JEE Main

MOCK TEST - 5

Instructions:

I. The JEE Main Test Paper consists of one paper containing 60 objective questions (four options with single correct answer) and 30 numerical value type questions from the syllabus of Physics, Chemistry and Mathematics.

II. The duration of paper would be 3 hours (180 minutes).

III. There will be total 90 questions : Physics - 30, Chemistry - 30, Mathematics - 30.

IV. Each question will carry 4 marks. For each correct response the applicant will be awarded four marks. For each incorrect answer there will be deduction of one mark.

V. There will be no negative marking for unattended questions. More than one answer of single question will also be considered as incorrect response and will be negatively marked.
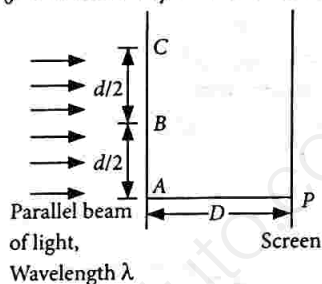
Max. Marks : 300 **Time : 180 minutes**

PHYSICS
Section-A (Multiple Choice Questions)

1. For the situation shown in figure below $BP - AP = \dfrac{\lambda}{3}$ and $D >> d$. The slits are of equal width, having intensity I_0. The intensity at P would be

Parallel beam of light, Wavelength λ

(a) $4I_0$ (b) $2I_0$ (c) $3I_0$ (d) $\dfrac{7}{2}I_0$

2. Surfaces of a thin equi-convex glass lens ($\mu = 1.5$) have radius of curvature R. Paraxial rays are incident on it. If the final image is formed at a distance $\dfrac{R}{13}$ from pole of the lens after n internal reflections. Then n is

(a) 2 (b) 3 (c) 4 (d) infinity

3. A circular disc A of radius r is made from an iron plate of thickness t and another circular disc B of radius $4r$ is made from an iron plate thickness $t/4$. The relation between the moments of inertia I_A and I_B is

(a) $I_A > I_B$ (b) $I_A = I_B$
(c) $I_A < I_B$
(d) depends on the actual values of t and r.

4. The rubber cord of catapult has a cross-sectional area 1 mm^2 and total unstretched length 10.0 cm. It is stretched to 12.0 cm and then released to project a missile of mass 5.0 g. Taking Young's modulus Y for rubber as 5.0×10^8 N m^{-2}, total elastic energy of catapult is converted into kinetic energy of missile without any heat loss. Calculate the velocity of projection (in m s^{-1}).

(a) 20 (b) 22 (c) 24 (d) 26

5. An electric field can deflect

(a) X-rays (b) neutrons
(c) α-particles (d) γ-rays

6. On heating one end of a rod, the temperature of the whole rod will be uniform when

(a) $K = 1$ (b) $K = 0$
(c) $K = 100$ (d) $K = \infty$

7. A large insulating thick sheet of thickness $2d$ carries a uniform charge per unit volume ρ. A particle of mass m, carrying a charge q having a sign opposite to that of the sheet is released from the surface of the sheet. The sheet does not offer any mechanical resistance to the motion of the particle. Find the oscillation frequency υ of the particle inside the sheet.

   (a) $\dfrac{1}{2\pi}\sqrt{\dfrac{q\rho}{m\varepsilon_0}}$
   (b) $\dfrac{1}{2\pi}\sqrt{\dfrac{2q\rho}{m\varepsilon_0}}$

   (c) $\dfrac{1}{4\pi}\sqrt{\dfrac{q\rho}{m\varepsilon_0}}$
   (d) $\dfrac{1}{4\pi}\sqrt{\dfrac{2q\rho}{m\varepsilon_0}}$

8. An inductive circuit draws a power 550 W from a 220 V–50 Hz source. The power factor of the circuit is 0.8. The current in the circuit lags behind the voltage. To bring its power factor to unity the capacitor connected in the circuit must have capacitance

   (a) $\dfrac{1}{8448\pi}$ F
   (b) 4224π F

   (c) 8448π F
   (d) $\dfrac{1}{4224\pi}$ F

9. When both jaws touch to each other, but zero mark of vernier scale is right to zero mark of main scale. Further 4th mark of vernier scale coincides with a certain mark of main scale. While measuring the side of a cube, it gives 10 divisions on main scale and 6th division of vernier scale coincide with main scale division. Find the side length of cube.
   (a) 10.6 mm
   (b) 11.0 mm
   (c) 10.2 mm
   (d) 10.4 mm

10. A body is moving up an inclined plane of angle θ with an initial kinetic energy K. The coefficient of friction between the plane and the body is μ. The work done against friction before the body comes to rest is

   (a) $\dfrac{\mu\cos\theta}{K\cos\theta+\sin\theta}$
   (b) $\mu K\cos\theta$

   (c) $\dfrac{\mu K\cos\theta}{\mu\cos\theta-\sin\theta}$
   (d) $\dfrac{\mu K\cos\theta}{\mu\cos\theta+\sin\theta}$

11. In figure shown, left arm of a U-tube is immersed in a hot water bath at temperature T, and right arm is immersed in a bath of melting ice, the height of manometric liquid in respective columns are h_T and h_0. Determine the coefficient of expansion of the liquid.

   (a) $\dfrac{1}{T}$
   (b) $\dfrac{h_T}{h_0 T}$
   (c) $\dfrac{h_T-h_0}{h_0 T}$
   (d) $\dfrac{h_T+h_0}{h_T T}$

12. The distance between plates of a parallel plate capacitor is $5d$. The positively charged plate is at $x = 0$ and negatively charged plate is at $x = 5d$. Two slabs one of conductor and the other of a dielectric of same thickness d are inserted between the plates as shown in figure. Potential (V) versus distance (x) graph will be
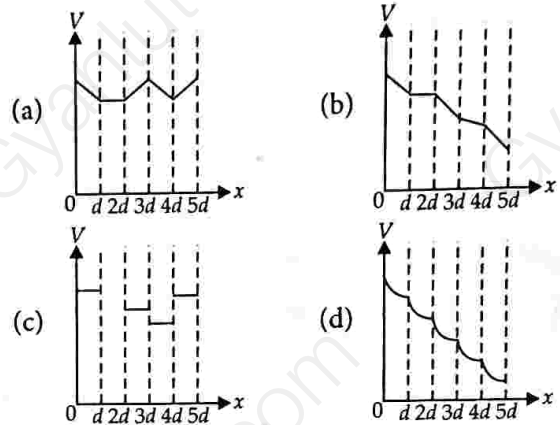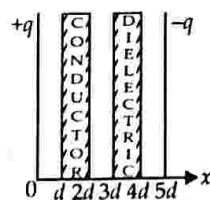
13. The frequency of a sonometer wire is υ, but when the weights producing the tensions are completely immersed in water the frequency becomes $\upsilon/2$ and on immersing the weights in a certain liquid the frequency becomes $\upsilon/3$. The specific gravity of the liquid is
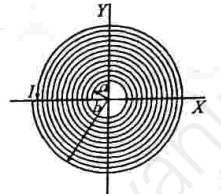
   (a) $\dfrac{4}{3}$
   (b) $\dfrac{16}{9}$
   (c) $\dfrac{15}{12}$
   (d) $\dfrac{32}{27}$

14. A particle is describing simple harmonic motion. If its velocities are v_1 and v_2 when the displacements from the mean position are y_1 and y_2 respectively, then its time period is

   (a) $2\pi\sqrt{\dfrac{y_1^2+y_2^2}{v_1^2+v_2^2}}$
   (b) $2\pi\sqrt{\dfrac{v_2^2-v_1^2}{y_1^2-y_2^2}}$

   (c) $2\pi\sqrt{\dfrac{v_1^2+v_2^2}{y_1^2+y_2^2}}$
   (d) $2\pi\sqrt{\dfrac{y_1^2-y_2^2}{v_2^2-v_1^2}}$

15. The mean distance between the atoms of iron is 3×10^{-10} m and interatomic force constant for iron is 7 N m^{-1}. The Young's modulus of elasticity for iron is
   (a) 2.33×10^5 N m^{-2}
   (b) 23.3×10^6 N m^{-2}
   (c) 2.33×10^9 N m^{-2}
   (d) 2.33×10^{10} N m^{-2}

16. In the circuit shown in figure, the current gain, $\beta = 100$ for the transistor. What would be the base resistance R_B so that $V_{CE} = 5$ V? (Neglect V_{BE}).
   (a) $2 \times 10^3\ \Omega$
   (b) $2 \times 10^5\ \Omega$
   (c) $1 \times 10^6\ \Omega$
   (d) $500\ \Omega$

17. A juggler throws balls into air. He throws one whenever the previous one is at its highest point. If he throws n balls each second, the height to which each ball will rise is

   (a) $\dfrac{g}{2n^2}$
   (b) $\dfrac{2g}{n^2}$
   (c) $\dfrac{2g}{n}$
   (d) $\dfrac{g}{4n^2}$

18. Four particles each of mass m are lying symmetrically on the rim of a disc of mass M and radius R. Moment of inertia of this system about an axis passing through one of the particle and perpendicular to plane of disc is

(a) $16mR^2$ (b) $(3M+16m)\dfrac{R^2}{2}$

(c) $(3M+12m)\dfrac{R^2}{2}$ (d) zero

19. A block of mass m is in contact with the cart C as shown in the figure. The coefficient of static friction between the block and the cart is μ. The acceleration α of the cart that will prevent the block from falling satisfies
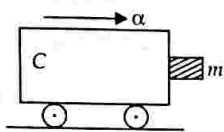
(a) $\alpha > \dfrac{mg}{\mu}$ (b) $\alpha > \dfrac{g}{\mu m}$

(c) $\alpha \geq \dfrac{g}{\mu}$ (d) $\alpha < \dfrac{g}{\mu}$

20. A cylindrical metallic rod in thermal contact with two reservoirs of heat at its two ends conducts an amount of heat Q in time t. The metallic rod is melted and the material is formed into a rod of half the radius of the original rod. What is the amount of heat conducted by the new rod, when placed in thermal contact with the two reservoirs in time t?

(a) $\dfrac{Q}{4}$ (b) $\dfrac{Q}{16}$ (c) $2Q$ (d) $\dfrac{Q}{2}$

Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

**21.** An electric circuit requires a total capacitance of $2\ \mu F$ across a potential of 1000 V. Large number of $1\ \mu F$ capacitances are available each of which would breakdown if the potential is more than 350 V. How many capacitors are required to make the circuit?

22. Interference fringes were produced in Young's double slit experiment using light of wavelength 5000 Å. When a film of material 2.5×10^{-3} cm thick was placed over one of the slits, the fringe pattern shifted by a distance equal to 20 fringe widths. The refractive index of the material of the film is _____.

23. Half-life of a radioactive substance is 20 minutes. Find the time (in minutes) between 20% and 80% decay.

24. A 10.0 g block with a charge of 8×10^{-5} C is placed in an electric field $\vec{E} = (3 \times 10^3)\,\hat{i} - 600\,\hat{j}$ N/C. If the block is released from rest at $t = 0$ from the origin, then its x-coordinates (in m) at $t = 3$ s is _____.

25. When the load on a wire is increased from 3 kg wt. to 5 kg wt the elongation increases from 0.61 mm to 1.02 mm. The work done during the extension of the wire is 16.023×10^{-x} J, where the value of x is _____.

26. In an electromagnetic wave, the amplitude of electric field is 1 V/m. The frequency of wave is 5×10^{14} Hz. The wave is propagating along z-axis. Then, the average energy density of electric field is given by $x \times 10^{-12}$ J/m^3 where the value of x is _____.

27. Two buses A and B are at positions 50 m and 100 m from the origin at time $t = 0$. They start moving in the same direction simultaneously with uniform velocity of 10 m s^{-1} and 5 m s^{-1}. The time (in s) at which A overtakes B, is _____.

28. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the object and eyepiece is 36 cm and the final image is formed at infinity. The focal length of the eyepiece (in cm) is _____.

29. A coil of self-inductance 0.16 H is connected to a condenser of capacity 0.81 μF. The frequency of AC that should be applied so that there is a resonance in the circuit is _____ cycles/second.

30. A body cools in 7 minutes from 60°C to 40°C. The temperature of the surroundings is 10°C. The temperature after the next 7 minutes will be _____ °C.

CHEMISTRY
Section-A (Multiple Choice Questions)

31. Which of the following orders is true regarding the basic nature of NH_2 group?
(a) o-Toluidine > Aniline > o-Nitroaniline
(b) o-Toluidine < Aniline > o-Nitroaniline
(c) o-Toluidine < Aniline < o-Nitroaniline
(d) o-Toluidine > Aniline < o-Nitroaniline

32. The substance magnetite is
(a) ferromagnetic   (b) diamagnetic
(c) antiferromagnetic   (d) ferrimagnetic.

33. When one mole of a gas is heated at constant volume, temperature is raised from 298 to 308 K. Heat supplied to gas is 500 J. Then which of the following is correct?
(a) $q = W = 500$ J, $\Delta U = 0$
(b) $q = \Delta U = 500$ J, $W = 0$
(c) $q = W = 500$ J, $\Delta U = 500$
(d) $\Delta U = 0$, $q = W = -500$ J

34. Pick the wrong statement from the following :
(a) Consumption of citrus fruits and green leafy vegetables in food prevents scurvy.
(b) Deficiency of vitamin B_6 (pyridoxine) results in convulsions.
(c) Sources of vitamin B_1 are yeast, milk, green vegetables and cereals.
(d) Deficiency of vitamin D causes xerophthalmia.

35. **Assertion :** The pressure exerted by a vapour of a liquid at a given temperature is called its vapour pressure.
Reason : If a non-volatile solute is added to a solvent to give a solution, the vapour pressure of the solution is found to be greater than the vapour pressure of the pure solvent.

(a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

(b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

(c) Assertion is correct statement but reason is wrong statement.

(d) Assertion is wrong statement but reason is correct statement.

36. The statement that is not correct is

(a) compressibility factor measures the deviation of real gas from ideal behaviour

(b) van der Waals' constant 'a' measures extent of intermolecular attractive forces for real gases

(c) critical temperature is the lowest temperature at which liquefaction of a gas first occurs

(d) Boyle point depends on the nature of real gas.

37. Total number of lone pairs of electrons in I_3^- ion is

(a) 3 (b) 6 (c) 9 (d) 12

38. The molar conductance of NaCl, HCl and CH_3COONa at infinite dilution are 126.45, 426.16 and 91.0 S cm^2 mol^{-1} respectively. The molar conductance of CH_3COOH at infinite dilution is

(a) 540.48 S cm^2 mol^{-1} (b) 201.28 S cm^2 mol^{-1}

(c) 390.71 S cm^2 mol^{-1} (d) 698.28 S cm^2 mol^{-1}

39. Orlon fibres are made up of

(a) cellulose (b) polyacrylonitrile

(c) polyamide (d) polyesters.

40. In the long form of the periodic table, the valence shell electronic configuration of $5s^2 5p^4$ corresponds to the element present in

(a) group 16 and period 6 (b) group 17 and period 5

(c) group 16 and period 5 (d) group 17 and period 6.

41. The correct order of intensity of colours of the compounds is

(a) $[Ni(H_2O)_6]^{2+} > [NiCl_4]^{2-} > [Ni(CN)_4]^{2-}$

(b) $[NiCl_4]^{2-} > [Ni(CN)_4]^{2-} > [Ni(H_2O)_6]^{2+}$

(c) $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

(d) $[Ni(CN)_4]^{2-} > [NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+}$

42. The first discovered antibiotic is

(a) streptomycin (b) penicillin

(c) chloramphenicol (d) tetracycline.

43. How many times oxyhaemoglobin is less stable than carboxyhaemoglobin?

(a) 50 (b) 200 (c) 500 (d) 300

44. Which of the following will most readily give the dehydrohalogenation product?

(a)   (b)

(c)   (d)
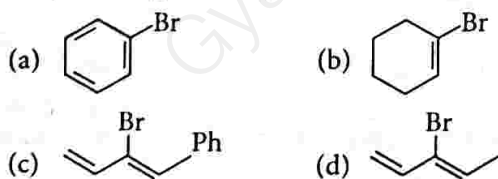
45. The intermediate during the addition of HCl to propene in the presence of peroxide is

(a) $CH_3\overset{+}{C}H_2CH_3$ (b) $CH_3\overset{+}{C}HCH_3$

(c) $CH_3CH_2CH_3$ (d) $CH_3CH_2\overset{+}{C}H_2$
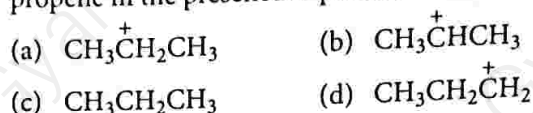
46. Match the following.

	Column-I		Column-II
(A)	Isotones	(i)	K-40 and Ca-40
(B)	Isotopes	(ii)	U-235 and Th-231
(C)	Isobars	(iii)	Protium and Tritium
(D)	Isodiaphers	(iv)	C-14 and O-16

(a) A – (i), B – (ii), C – (iii), D – (iv)

(b) A – (iv), B – (iii), C – (i), D – (ii)

(c) A – (iii), B – (ii), C – (iv), D – (i)

(d) A – (ii), B – (iii), C – (i), D – (iv)

47. Assertion : Rate of reaction of alkyl halide in Williamson's synthesis reaction is $1°RX > 2°RX > 3°RX$.

Reason : It is a type of bimolecular substitution reaction (S_N2).

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false.

(d) If both assertion and reason are false.

48. Assertion : One mole of NaCl contains 6.023×10^{23} formula unit of sodium chloride.

Reason : 58.5 g of NaCl also contains 6.023×10^{23} formula unit of NaCl.

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false.

(d) If both assertion and reason are false.

49. Which of the following compounds will show highest dipole moment?

(I) (II) (III) (IV)
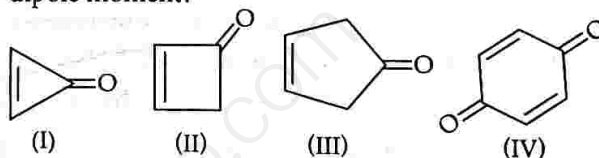
(a) (I) (b) (III) (c) (II) (d) (IV)

50. The major product in the following reaction is

(a)   (b)

(c)   (d)
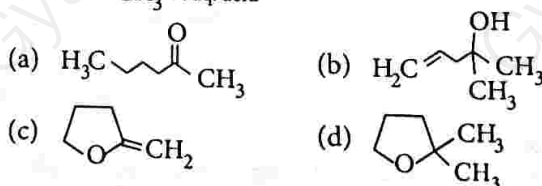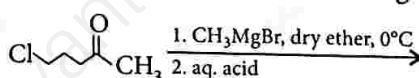
Attempt any 5 questions out of 10

51. The pH at which $Mg(OH)_2$ begins to precipitate from a solution containing 0.1 M Mg^{2+} ions is _____. (K_{sp} for $Mg(OH)_2 = 1.0 \times 10^{-11}$)

52. $2A + B_2 \rightarrow 2AB$ is an elementary reaction. For a certain quantity of reactants, if the volume of the reaction vessel is reduced by a factor of 2, then rate of the reaction increases by a factor of _____. (Round off to the Nearest Integer)

53. An organic compound ($C_8H_{10}O_2$) rotates plane-polarized light. It produces pink colour with neutral $FeCl_3$ solution. Total number of all the possible isomers for this compound is _____.

54. A plot of log (x/m) (y-axis) and log p (x-axis) gives a straight line for Freundlich adsorption isotherm the intercept and slope for the line is 0.3771 and 3, respectively. The mass of gas, absorbed per gram of adsorbent if the initial pressure is 0.04 atm, is _____ $\times 10^{-5}$ g. (log 3 = 0.4771)

55. Commercial sample of H_2O_2 is labelled as 10 V. Its % strength is _____.

56. (a) Cinnabar, (b) Argentite, (c) Zinc blende and (d) Copper pyrites. How many of these minerals are sulphide based?

57. The number of covalent compounds among the following is/are _____.
NaCl, KCl, $BeCl_2$, $MgCl_2$, $CaCl_2$

58. Among the following, the number of compounds that can react with PCl_5 to give $POCl_3$ is _____.
O_2, CO_2, SO_2, H_2O, H_2SO_4, P_4O_{10}

59. In borax, the number of $B-O-B$ links present is _____.

60. The disproportionation of MnO_4^{2-} in acidic medium resulted in the formation of MnO_2, then the spin-only magnetic moment (μ) value of MnO_2 in B.M. is _____. (Nearest integer)

MATHEMATICS
Section-A (Multiple Choice Questions)

61. The variate of a distribution takes the values 1, 2, 3, ..., n with frequencies n, $n - 1$, $n - 2$, ..., 3, 2, 1, then mean value of the distribution is
(a) $\dfrac{n(n+2)}{3}$
(b) $\dfrac{n(n+1)(n+2)}{6}$
(c) $\dfrac{n+2}{3}$
(d) $\dfrac{(n+1)(n+2)}{6}$

62. The probability of simultaneous occurrence of atleast one of two events A and B is p. If the probability that exactly one of A, B occurs is q, then $P(A') + P(B') =$
(a) $1 - p + 2q$
(b) $2 - 2p + q$
(c) $p - q$
(d) None of these

63. If matrix A is a circulant matrix whose elements of first row are a, b, $c > 0$ such that $abc = 1$ and $A^T A = I$, then $a^3 + b^3 + c^3$ equals to
(a) 0
(b) 4
(c) 1
(d) 3

64. If p is the length of the perpendicular from the origin to the line whose intercepts with the coordinate axes are $\dfrac{1}{3}$ and $\dfrac{1}{4}$, then the value of p is
(a) 3/4
(b) 1/12
(c) 5
(d) 1/5

65. $\lim\limits_{x \to \infty} x \sin\left(\dfrac{2}{x}\right)$ is equal to
(a) 2
(b) 1/2
(c) ∞
(d) 0

66. If $\dfrac{\left[\sin\dfrac{\theta}{2} + \cos\dfrac{\theta}{2}\right] - i\cot\dfrac{\theta}{2}}{1 + 2i\cos\dfrac{\theta}{2}}$ is real, then $\theta =$
(a) $\dfrac{n\pi}{2}$
(b) $n\pi$
(c) $\dfrac{n\pi}{4}$
(d) $2n\pi$

67. The number of n digit numbers, which contain the digits 3 and 6, but not the digit 0, 1, 8, 9 is
(a) $6^n - 2\cdot5^n + 4^n$
(b) $6^n + 2\cdot5^n - 4^n$
(c) $6^n - 4^n$
(d) None of these

68. Let $f(x) = \dfrac{x-1}{x+1}$, $x \in R - \{0, -1, 1\}$.
If $f^{n+1}(x) = f(f^n(x))$ for all $n \in N$, then $f^6(6) + f^7(7)$ is equal to
(a) $\dfrac{7}{6}$
(b) $-\dfrac{3}{2}$
(c) $\dfrac{7}{12}$
(d) $-\dfrac{11}{12}$

69. If the function $f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \le \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is twice differentiable, then the ordered pair (k_1, k_2) is equal to
(a) $\left(\dfrac{1}{2}, 1\right)$
(b) $(1, 0)$
(c) $\left(\dfrac{1}{2}, -1\right)$
(d) $(1, 1)$

70. Let the volume of a parallelopiped whose coterminous edges are given by $\vec{\alpha} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{\beta} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{\gamma} = 2\hat{i} + \hat{j} + \hat{k}$ be 2 cu. unit. If θ be the angle between the edges $\vec{\alpha}$ and $\vec{\gamma}$, then $\cos\theta$ can be
(a) $\dfrac{5}{3\sqrt{2}}$
(b) $\dfrac{8}{3\sqrt{2}}$
(c) $\dfrac{8}{9\sqrt{2}}$
(d) $\dfrac{4}{3}$

71. Let $f(x) = |[x] x|$ for $-1 \le x \le 2$, then number of points where $f(x)$ is discontinuous are

(a) 1   (b) 2   (c) 3   (d) 4

72. If a function $y = f(x)$ is such that $f(1) = 1$ and
$$x\int_0^x (1-z)f(z)dz = \int_0^x z\, f(z)dz,$$ then $f(x)$ equals

(a) $\dfrac{e^{1+\frac{1}{x}}}{x^3}$ (b) $\dfrac{x^3}{e^{1-1/x}}$

(c) $\dfrac{e^{x-1/x}}{x^3}$ (d) $\dfrac{e^{1-1/x}}{x^3}$

73. A vertical pole of height h is placed at a point on an inclined plane of angle $15°$ at a distance 'h' from its base. When angle of elevation of the sun is $30°$, the shadow of pole falling along the line of the greatest slope just reaches a point which is at a distance 'd' away from the base of the plane, then $\dfrac{d}{h}$ equals

(a) $\dfrac{4\sqrt{6}-3-\sqrt{2}}{4\sqrt{2}}$ (b) $\dfrac{4\sqrt{6}+3+\sqrt{3}}{4\sqrt{2}}$

(c) $\dfrac{4\sqrt{6}+3-\sqrt{3}}{4\sqrt{2}}$ (d) None of these

74. Which of the following statements is a tautology?

(a) $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
(b) $(\sim p \wedge q) \wedge (q \rightarrow p)$
(c) $(p \wedge q) \vee (p \wedge r)$
(d) $(p \vee q) \vee r \leftrightarrow p \vee (q \vee r)$

75. The locus of the orthocentre of the triangle formed by the lines $(1 + p)\, x - py + p\,(1 + p) = 0$, $(1 + q)\, x - qy + q\,(1 + q) = 0$, and $y = 0$, $p \ne q$ is

(a) hyperbola   (b) parabola
(c) ellipse   (d) straight line

76. Let F be the family of ellipses whose centre is the origin and major axis is the y-axis. Then the differential equation of family F is

(a) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx}\left(x\dfrac{dy}{dx} - y\right) = 0$

(b) $xy\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx}\left(x\dfrac{dy}{dx} - y\right) = 0$

(c) $xy\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx}\left(x\dfrac{dy}{dx} - y\right) = 0$

(d) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx}\left(x\dfrac{dy}{dx} - y\right) = 0$
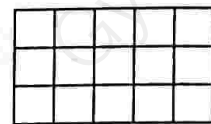
77. In $[0, 1]$, Lagrange's Mean Value Theorem is not applicable to

(a) $f(x) = \begin{cases} \dfrac{1}{4} - x & , \ x < \dfrac{1}{4} \\ \left(\dfrac{1}{4} - x\right)^2 & , \ x \ge \dfrac{1}{4} \end{cases}$

(b) $f(x) = \begin{cases} \dfrac{\sin x}{x} & , \ x \ne 0 \\ 1 & , \ x = 0 \end{cases}$

(c) $f(x) = x\,|x|$

(d) $f(x) = |x|$

78. A rectangle with sides $2m - 1$, $2n - 1$ is divided into squares of unit length by drawing parallel lines as shown in the diagram.

    The number of rectangles with odd side length is

(a) $(m + n + 1)^2$
(b) $mn\,(m + 1)\,(n + 1)$
(c) $m^2 n^2$
(d) 4^{m+n-1}

79. The sphere of constant radius $2k$ passes through $O(0,0,0)$ and meet the axes at A, B, C. The locus of the centroid of tetrahedron $OABC$ is $x^2 + y^2 + z^2 = \lambda k^2$, then the value of λ equals

(a) 2   (b) 3
(c) 4   (d) 1

80. Let $g(x) = \displaystyle\int_{3-x}^{3+x} t\,|f'(t)|\,dt$, where $f(x)$ does not behave like a constant function in any interval (a, b) and the graph of $f'(x)$ is symmetric about the line $x = 3$. Then

(a) $g(x)$ is increasing $\forall x \in R$
(b) $g(x)$ is increasing only if $x < 3$
(c) $g(x)$ is increasing only if $f(x)$ is increasing
(d) $g(x)$ is decreasing $\forall x \in R$

Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

81. The number of real tangents that can be drawn to the ellipse $3x^2 + 5y^2 = 32$ passing through $(3, 5)$ is _____.

82. If the three equations $x^2 + px + 12 = 0$, $x^2 + qx + 15 = 0$ and $x^2 + (p + q)\,x + 36 = 0$ have a common possible root, then the value of $p^2 + q^2$ is _____.

83. The coefficients of x^p is 28 in the expansion of $(1 + x)^{p+q}$, then coefficient of $\dfrac{x^q}{x^p}$ is _____.

84. The sum to infinity of the series $1 + \dfrac{2}{3} + \dfrac{6}{3^2} + \dfrac{10}{3^3} + \dfrac{14}{3^4} + \dots$ is _____.

85. Let $T > 0$ be a fixed real number. Suppose f is a continuous function such that for all $x \in R, f(x + T) = f(x)$. If $I_1 = \int_0^T f(x)\, dx$ and $I_2 = \int_3^{3+3T} f(2x)\, dx$. Then the value of $\dfrac{I_2}{I_1}$ is _____.

86. An NGO conducted a survey of 2500 consumers and reported that 1720 consumers liked product A and 1450 consumers liked product B. Then the least number that must have liked both the products is _____.

87. Sum of how many terms of the sequence $\cot^{-1} 3 + \cot^{-1} 7 + \cot^{-1} 13 + \cot^{-1} 21....$ is $\dfrac{1}{2}\cos^{-1}\left(\dfrac{24}{145}\right)$?

88. The number of integral values of α for which the point $(\alpha - 1, \alpha + 1)$ lies in the larger segment of the circle $x^2 + y^2 - x - y - 6 = 0$ made by the chord whose equation is $x + y - 2 = 0$ is _____.

89. A peon has eight master keys to open several new rooms. Only one master key will open any given room. If 40% of these rooms are usually left unlocked, then the probability that the peon gets into specific room if he selects three master keys at random is $\dfrac{5}{p}$, where p is _____.

90. From the point $P(5, 2)$ normals are drawn to the parabola $y^2 = 4x$ to meet the curve at A and B. If the tangents to the curve at A and B meet at the point Q, then $\left(\dfrac{AB}{PQ}\right)^2$ is equal to _____.

SPACE FOR ROUGH WORK