

# JEE Main

## MOCK TEST - 4

### Instructions:

- I. The JEE Main Test Paper consists of one paper containing 60 objective questions (four options with single correct answer) and 30 numerical value type questions from the syllabus of Physics, Chemistry and Mathematics.
- II. The duration of paper would be 3 hours (180 minutes).
- III. There will be total 90 questions : Physics - 30, Chemistry - 30, Mathematics - 30.
- IV. Each question will carry 4 marks. For each correct response the applicant will be awarded four marks. For each incorrect answer there will be deduction of one mark.
- V. There will be no negative marking for unattended questions. More than one answer of single question will also be considered as incorrect response and will be negatively marked.

Max. Marks : 300

Time : 180 minutes

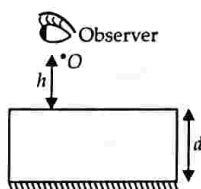
### PHYSICS

#### Section-A (Multiple Choice Questions)

1. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0°C. What is the change in the diameter of the hole when the sheet is heated to 227 °C? Coefficient of linear expansion of copper is  $1.70 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ .

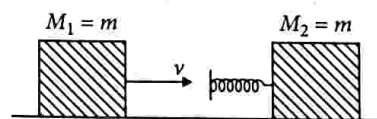
- (a)  $1.44 \times 10^{-2} \text{ cm}$       (b)  $2.44 \times 10^{-3} \text{ cm}$   
(c)  $1.44 \times 10^{-2} \text{ mm}$       (d)  $2.44 \times 10^{-3} \text{ mm}$

2. A point luminous object (O) is at a distance  $h$  from front face of a glass slab of width  $d$  and of refractive index  $\mu$ . On the back face of slab there is a reflecting plane mirror. An observer sees the image of object in mirror [figure]. Distance of image from front face as seen by observer will be



- (a)  $h + \frac{2d}{\mu}$       (b)  $2h + 2d$   
(c)  $h + d$       (d)  $h + \frac{d}{\mu}$

3. Two blocks  $M_1$  and  $M_2$  having equal mass are to move on a horizontal frictionless surface.  $M_2$  is attached to a massless spring as shown in figure. Initially  $M_2$  is at rest and  $M_1$  is moving toward  $M_2$  with speed  $v$  and collides head-on with  $M_2$ .



- (a) While spring is fully compressed, all the kinetic energy of  $M_1$  is stored as potential energy of spring.  
(b) While spring is fully compressed the system momentum is not conserved, though final momentum is equal to initial momentum.  
(c) If spring is massless, the final state of the  $M_2$  is state of rest.  
(d) If the surface on which blocks are moving has friction, then collision cannot be elastic.
4. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the

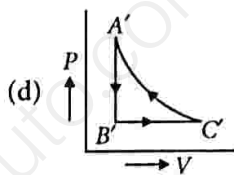
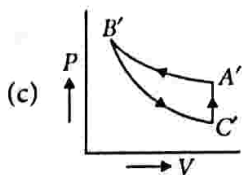
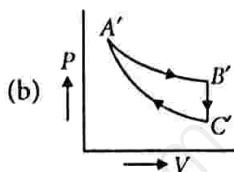
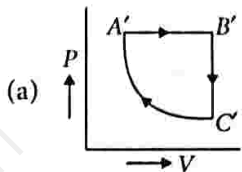
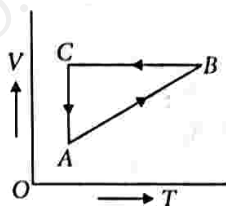
main scale is 2.5 mm and that on the circular scale is 20 divisions.

If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is  
 (a) 0.9% (b) 2.4% (c) 3.1% (d) 4.2%

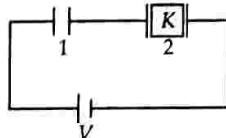
5. A carpet of mass  $M$ , made of an extensible material is rolled along its length in the form of a cylinder of radius  $R$  and kept on a rough floor. If the carpet is unrolled, without sliding to a radius  $R/2$ , the decrease in potential energy is

- (a)  $\frac{1}{2}MgR$  (b)  $\frac{7}{8}MgR$  (c)  $\frac{5}{8}MgR$  (d)  $\frac{3}{4}MgR$

6. A cyclic process  $ABCA$  shown in  $V-T$  diagram, is performed with a constant mass of an ideal gas. Which of the following graphs in figure represents the corresponding process on a  $P-V$  diagram?



7. Two identical capacitors 1 and 2 are connected in series to a battery as shown in figure. Capacitor 2 contains a dielectric slab of dielectric constant  $K$  as shown.  $Q_1$  and  $Q_2$  are the charges stored in the capacitors. Now the dielectric slab is removed and the corresponding charges are  $Q'_1$  and  $Q'_2$ . Then



- (a)  $\frac{Q'_1}{Q_1} = \frac{K+1}{K}$  (b)  $\frac{Q'_2}{Q_2} = \frac{K+1}{2}$   
 (c)  $\frac{Q'_2}{Q_2} = \frac{K+1}{2K}$  (d)  $\frac{Q'_1}{Q_1} = \frac{K}{2}$

8. Two pendulums differ in lengths by 22 cm. They oscillate at the same place so that one of them makes 30 oscillations and the other makes 36 oscillations during the same time. The lengths (in cm) of the pendulums are  
 (a) 72 and 50 (b) 60 and 38  
 (c) 50 and 28 (d) 80 and 58

9. A galvanometer of  $50\Omega$  resistance has 25 divisions. A current of  $4 \times 10^{-4}$  A gives a deflection of one division.

To convert this galvanometer into a voltmeter having a range of 25 V, it should be connected with a resistance of

- (a)  $2500\Omega$  as a shunt (b)  $2450\Omega$  as a shunt  
 (c)  $2550\Omega$  in series (d)  $2450\Omega$  in series.

10. A boy of mass 30 kg starts running from rest along a circular path of radius 6 m with constant tangential acceleration of magnitude  $2 \text{ m s}^{-2}$ . After 2 s from start he feels that his shoes started slipping on ground. The friction between his shoes and ground is (Take  $g = 10 \text{ m s}^{-2}$ )

- (a)  $1/2$  (b)  $1/3$  (c)  $1/4$  (d)  $1/5$

11. Two short bar magnets of magnetic moment  $M$  each are arranged at the opposite corners of a square of side  $d$  such that their centres coincide with the corners and their axes are parallel. If the like poles are in the same direction, the magnitude of the magnetic induction at any of the other corners of the square is

- (a)  $\frac{\mu_0 M}{4\pi d^3}$  (b)  $\frac{\mu_0 2M}{4\pi d^3}$  (c)  $\frac{\mu_0 M}{4\pi 2d^3}$  (d)  $\frac{\mu_0 M^3}{4\pi 2d^3}$

12. In the given figure the pulley is assumed massless and frictionless. If the friction force on the object of mass  $m$  is  $f$ , then its acceleration in terms of the force  $F$  will be equal to

- (a)  $(F-f)/m$  (b)  $\left(\frac{F}{2} - f\right)/m$   
 (c)  $F/m$  (d) none of these

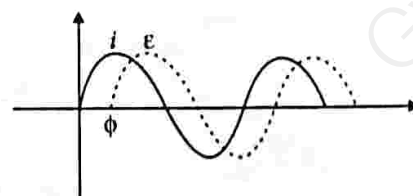
13. A particle executes SHM with an amplitude of 2 cm. When the particle is at 1 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is

- (a)  $\frac{1}{2\pi\sqrt{3}}$  (b)  $2\pi\sqrt{3}$  (c)  $\frac{2\pi}{\sqrt{3}}$  (d)  $\frac{\sqrt{3}}{2\pi}$

14. The Poisson's ratio of a material is 0.4. If a force is applied to a wire of this material, there is a decrease of cross-sectional area by 2%. The percentage increase in its length is

- (a) 3% (b) 2.5% (c) 1% (d) 0.5%

15. When an AC source of emf  $\epsilon = \epsilon_0 \sin(100t)$  is connected across a circuit, the phase difference between emf ( $\epsilon$ ) and current ( $i$ ) in the circuit is observed to be  $\pi/4$ , as shown in figure. If the circuit consist possibly only of  $RC$  or  $RL$  in series, find the resistance, capacitance or inductance in the circuit.



- (a)  $R = 1 \text{ k}\Omega$ ,  $C = 5 \mu\text{F}$  (b)  $R = 1 \text{ k}\Omega$ ,  $C = 10 \mu\text{F}$   
 (c)  $R = 1 \text{ k}\Omega$ ,  $C = 1 \text{ F}$  (d)  $R = 1 \text{ k}\Omega$ ,  $L = 10 \text{ H}$

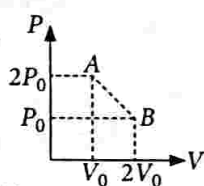
16. The radius of a planet is twice the radius of earth. Both have almost equal average mass-densities.  $V_P$  and  $V_E$  are escape velocities of the planet and the earth, respectively, then

- (a)  $V_P = 1.5 V_E$  (b)  $V_P = 2 V_E$   
 (c)  $V_E = 3 V_P$  (d)  $V_E = 1.5 V_P$

17. What is the minimum thickness of a thin film required for constructive interference in the reflected light from it? Given, the refractive index of the film = 1.5, wavelength of the light incident on the film = 600 nm.

- (a) 100 nm (b) 300 nm  
 (c) 50 nm (d) 200 nm

18.  $n$  mole of an ideal gas undergoes a process  $A \rightarrow B$  as shown in the given figure. Maximum temperature of the gas during process is

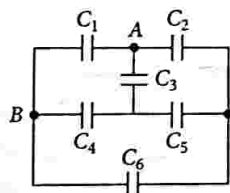


- (a)  $\frac{3P_0V_0}{2nR}$  (b)  $\frac{9P_0V_0}{4nR}$   
 (c)  $\frac{9P_0V_0}{2nR}$  (d)  $\frac{9P_0V_0}{nR}$

19. A particle of mass  $m$  moves in a one-dimensional potential energy  $U(x) = -ax^2 + bx^4$ , where  $a$  and  $b$  are positive constants. The angular frequency of small oscillations about the minima of the potential energy is equal to

- (a)  $\pi\sqrt{\frac{a}{2b}}$  (b)  $2\sqrt{\frac{a}{m}}$  (c)  $\sqrt{\frac{2a}{m}}$  (d)  $\sqrt{\frac{a}{2m}}$

20. In the network shown in figure, all the capacitors are identical. When an emf is applied across A, B one of the capacitors remains uncharged. That capacitor is



- (a)  $C_1$  (b)  $C_2$  (c)  $C_5$  (d)  $C_6$

### Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

21. On a foggy day two drivers spot each other when they are just 80 metres apart. They are travelling at  $72 \text{ km h}^{-1}$  and  $60 \text{ km h}^{-1}$ , respectively. Both of them applied brakes retarding their cars at the rate of  $5 \text{ m s}^{-2}$ . The total distance (in m) covered by the two cars is \_\_\_\_\_.
22. A short magnet is placed in the magnetic meridian with its South Pole pointing north. The null point is obtained 20 cm away from the centre of the magnet. If  $B_H = 30 \mu\text{T}$  the magnetic moment of the magnet (in  $\text{A m}^2$ ) is \_\_\_\_\_.

23. A rectangular tank is 10 m long, 5 m broad and 3 m high. It is filled to the rim with water of density  $10^3 \text{ kg m}^{-3}$ . The thrust at the bottom and walls of the tank due to hydrostatic pressure is  $2.793 \times 10^x \text{ N}$ , where the value of  $x$  is \_\_\_\_\_.

24. The oxygen molecule has a mass of  $5.30 \times 10^{-26} \text{ kg}$  and a moment of inertia of  $1.94 \times 10^{-46} \text{ kg m}^2$  about an axis through its centre perpendicular to the line joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two third of its kinetic energy of translation. The average angular velocity of the molecule is  $x \times 10^{12} \text{ rad s}^{-1}$ , where the value of  $x$  is \_\_\_\_\_.

25. A wheel of moment of inertia  $2.5 \text{ kg m}^2$  has initial angular velocity of  $40 \text{ rad s}^{-1}$ . A constant torque of  $10 \text{ N m}$  acts on the wheel. The time (in s) during which the wheel is accelerated to  $60 \text{ rad s}^{-1}$  is \_\_\_\_\_.

26. A capacitor of capacitance  $5 \mu\text{F}$  is charged to potential 20 V and then isolated. Now, an uncharged capacitor is connected in parallel to it. If the charge distributes equally on these capacitors, then the total energy (in  $\mu\text{J}$ ) stored in capacitors is \_\_\_\_\_.

27. In  $n$ - $p$ - $n$  transistor circuit, the collector current is 10 mA. If 95% of the electrons emitted reach the collector, then the base current (in mA) is \_\_\_\_\_.

28. Two beams of light having intensities  $I$  and  $4I$  interfere to produce a fringe pattern on a screen. The phase difference between the beams is  $\pi/2$  at point A and  $2\pi$  at point B. Then, the difference between the resultant intensity between A and B is  $nI$  where the value of  $n$  is \_\_\_\_\_.

29. A bulb with power one watt have efficiency of 10%. If the wavelength of light emitted by the bulb is 500 nm, then the number of photons emitted by it is  $x \times 10^{17}$  where the value of  $x$  is \_\_\_\_\_.

30. A progressive wave is represented by  $y = 5 \sin(100\pi t - 2\pi x)$ , where  $x$  and  $y$  are in m and  $t$  is in s. The maximum particle velocity given by  $x\pi \text{ m s}^{-1}$  where the value of  $x$  is \_\_\_\_\_.

## CHEMISTRY

### Section-A (Multiple Choice Questions)

31. The chemical reaction,  $2\text{O}_3 \rightarrow 3\text{O}_2$  proceeds as follows:



The rate law expression should be

- (a)  $r = k [\text{O}_3]^2$  (b)  $r = k [\text{O}_3]^2 [\text{O}_2]^{-1}$   
 (c)  $r = k [\text{O}_3] [\text{O}_2]$  (d) unpredictable.

32. 5 L aqueous solution is kept in the presence of oxygen and suitable microorganism for five days at 20 °C. If the O<sub>2</sub> consumed is 0.2 g, the BOD value of the sample is

- (a) 4 ppm (b) 0.4 ppm  
(c) 40 ppm (d) 20 ppm

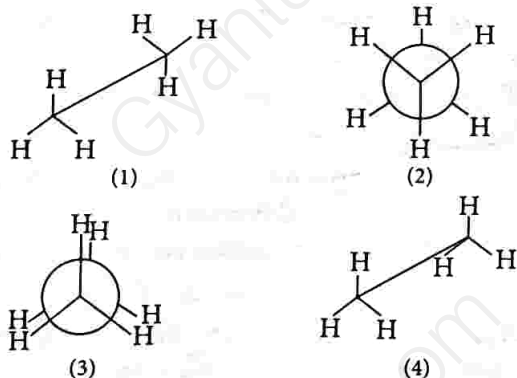
33. The dispersed phase in colloidal iron (III) hydroxide and colloidal gold is positively and negatively charged respectively. Which of the following statements is not correct?

- (a) Magnesium chloride solution coagulates gold sol readily than iron (III) hydroxide sol.  
(b) Sodium sulphate solution causes coagulation in both sols.  
(c) Mixing of the two sols has no effect.  
(d) Coagulation in both sols can be brought about by electrophoresis.

34. Which of the following pairs of isomers is not correctly matched with its type of isomerism?

- (a) [Co(NH<sub>3</sub>)<sub>6</sub>] [Cr(CN)<sub>6</sub>] and [Cr(NH<sub>3</sub>)<sub>6</sub>] [Co(CN)<sub>6</sub>]  
– Coordination isomerism  
(b) [Co(NH<sub>3</sub>)<sub>5</sub>NO<sub>2</sub>]Cl<sub>2</sub> and [Co(NH<sub>3</sub>)<sub>5</sub>ONO]Cl<sub>2</sub>  
– Linkage isomerism  
(c) [Co(py)<sub>2</sub>(H<sub>2</sub>O)<sub>2</sub>Cl<sub>2</sub>]Cl and [Co(py)<sub>2</sub>(H<sub>2</sub>O)Cl<sub>3</sub>]H<sub>2</sub>O  
– Coordination isomerism  
(d) [Pt(NH<sub>3</sub>)<sub>4</sub>Br<sub>2</sub>]Cl<sub>2</sub> and [Pt(NH<sub>3</sub>)<sub>4</sub>Cl<sub>2</sub>]Br<sub>2</sub>  
– Ionisation isomerism

35. In the following structures which two forms are the staggered conformation of ethane?

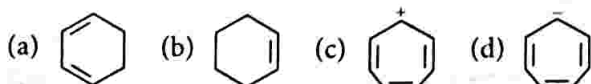


- (a) 1 and 4 (b) 2 and 3  
(c) 1 and 2 (d) 1 and 3

36. Extraction of zinc from zinc blende is achieved by

- (a) electrolytic reduction  
(b) roasting followed by reduction with carbon  
(c) roasting followed by reduction with another metal  
(d) roasting followed by self-reduction.

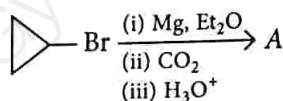
37. Which of the following is aromatic?



38. The species having no S—S bond is

- (a) S<sub>2</sub>O<sub>4</sub><sup>2-</sup> (b) S<sub>2</sub>O<sub>3</sub><sup>2-</sup> (c) S<sub>2</sub>O<sub>7</sub><sup>2-</sup> (d) S<sub>2</sub>O<sub>5</sub><sup>2-</sup>

39. For the following reaction, the product 'A' is



- (a) COOH (b) (COO)<sub>2</sub>Mg  
(c) C=O (d)

40. (CH<sub>3</sub>)<sub>3</sub>SiCl is used during polymerisation of organosilicons because

- (a) the chain length of organosilicon polymers can be controlled by adding (CH<sub>3</sub>)<sub>3</sub>SiCl  
(b) (CH<sub>3</sub>)<sub>3</sub>SiCl improves the quality and yield of the polymer  
(c) (CH<sub>3</sub>)<sub>3</sub>SiCl does not block the end terminal of silicone polymer  
(d) (CH<sub>3</sub>)<sub>3</sub>SiCl acts as a catalyst during polymerisation.

41. The correct formula of permanganic acid is

- (a) HMnO<sub>4</sub> (b) HMnO<sub>5</sub>  
(c) H<sub>2</sub>MnO<sub>4</sub> (d) H<sub>2</sub>MnO<sub>3</sub>

42. **Assertion :** Alkali metals impart colour to the flame.

**Reason :** Their ionisation energies are low.

- (a) Assertion and reason both are correct and reason is correct explanation for assertion.  
(b) Assertion and reason both are correct but reason is not correct explanation for assertion.  
(c) Assertion is correct but reason is wrong.  
(d) Assertion is wrong but reason is correct.

43. Among the following which is least acidic?

- (a) Phenol (b) *o*-Cresol  
(c) *p*-Nitrophenol (d) *p*-Chlorophenol

44. Consider the following statements :

- I. Atomic hydrogen is obtained by molecular hydrogen on a hot filament of tungsten or platinum.  
II. Hydrogen gas will not reduce heated aluminium oxide.  
III. Finely divided palladium adsorbs large volume of hydrogen gas.  
IV. Nascent hydrogen is less active than ordinary hydrogen.

Which of the above statements are correct?

- (a) Only I and II (b) Only II and IV  
(c) Only I, II and III (d) Only I and IV

45. The carboxyl functional group (–COOH) is present in

- (a) picric acid (b) barbituric acid  
(c) ascorbic acid (d) aspirin.

46. 0.24 g of a volatile substance displaced 53.78 mL of air at STP. The molecular weight of the substance is  
 (a) 24 g (b) 53.78 g  
 (c) 50 g (d) 100 g
47. During the preparation of arenediazonium salts, the excess of nitrous acid is destroyed by adding  
 (a) aq. NaOH (b) aq. Na<sub>2</sub>CO<sub>3</sub>  
 (c) aq. NH<sub>2</sub>CONH<sub>2</sub> (d) aq. KI
48. A solution is 0.1 M with respect to Ag<sup>+</sup>, Ca<sup>2+</sup>, Mg<sup>2+</sup> and Al<sup>3+</sup>. Which will precipitate at lowest concentration of PO<sub>4</sub><sup>3-</sup> ion when Na<sub>3</sub>PO<sub>4</sub> solution is added?  
 (a) Ag<sub>3</sub>PO<sub>4</sub> ( $K_{sp} = 1 \times 10^{-6}$ )  
 (b) Ca<sub>3</sub>(PO<sub>4</sub>)<sub>2</sub> ( $K_{sp} = 1 \times 10^{-33}$ )  
 (c) Mg<sub>3</sub>(PO<sub>4</sub>)<sub>2</sub> ( $K_{sp} = 1 \times 10^{-24}$ )  
 (d) AlPO<sub>4</sub> ( $K_{sp} = 1 \times 10^{-20}$ )
49. Elastol is a polymer used to cleanup oil spill. It is a non-toxic, non-dispersant chemical. One gallon can remove 150 gallons of heavy oil. The monomer of elastol is  
 (a) CH<sub>2</sub>=CH-CN (b) CH<sub>2</sub>=C(CH<sub>3</sub>)<sub>2</sub>  
 (c) CH<sub>2</sub>=C(CH<sub>3</sub>)-COOCH<sub>3</sub> (d) CH<sub>2</sub>=CH-Ph

50. Match the column I with column II and select the correct option.

Column-I		Column-II	
P.	Presence of halogen	1.	HNO <sub>3</sub> /AgNO <sub>3</sub>
Q.	Presence of sulphur	2.	(NH <sub>4</sub> ) <sub>2</sub> MoO <sub>4</sub>
R.	Presence of nitrogen	3.	Na <sub>2</sub> [Fe(CN) <sub>5</sub> NO]
S.	Presence of phosphorous	4.	FeSO <sub>4</sub> /dil. H <sub>2</sub> SO <sub>4</sub>

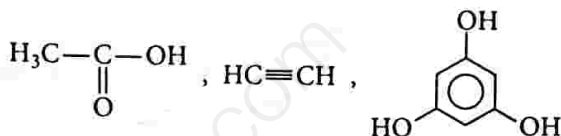
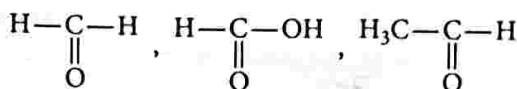
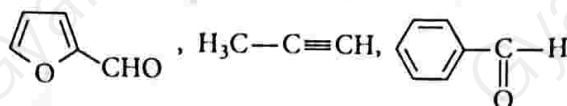
- |     |   |   |   |   |
|-----|---|---|---|---|
|     | P | Q | R | S |
| (a) | 1 | 3 | 4 | 2 |
| (b) | 2 | 4 | 1 | 3 |
| (c) | 1 | 2 | 3 | 4 |
| (d) | 4 | 3 | 2 | 1 |

### Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

51. The number of disulphide linkages present in insulin is \_\_\_\_\_.
52. An accelerated electron has a speed of  $6 \times 10^6$  m s<sup>-1</sup> with an uncertainty of 0.04%. The uncertainty in finding its location while in motion is  $x \times 10^{-9}$  m. The value of  $x$  is \_\_\_\_\_.  
 [Use mass of electron =  $9.1 \times 10^{-31}$  kg,  $h = 6.63 \times 10^{-34}$  J s,  $\pi = 3.14$ ]

53. Among the following, how many of them will give positive test with Tollens' reagent?



54. The number of the pairs containing elements with similar atomic radii is \_\_\_\_\_.  
 (I) Co, Ni (II) Zn, Mo  
 (III) Rh, Ir (IV) Hf, Ti
55. If KBr is doped with  $10^{-6}$  mole percent of SrBr<sub>2</sub>, then the number of cationic vacancies in 2 g of KBr crystal is \_\_\_\_\_  $\times 10^{14}$ .  
 [Atomic mass : K : 39.1 u, Br : 79.9 u,  $N_A = 6.023 \times 10^{23}$ ]
56. Consider the following species :  
 NO<sub>3</sub><sup>-</sup>, SO<sub>4</sub><sup>2-</sup>, ClO<sub>3</sub><sup>-</sup>, SO<sub>3</sub>, PO<sub>4</sub><sup>3-</sup>, XeO<sub>3</sub>, CO<sub>3</sub><sup>2-</sup>, SO<sub>3</sub><sup>2-</sup>  
 Then, calculate value of  $|x - y|$ , where  
 $x$  : Total number of species which have bond order 1.5 or greater than 1.5.  
 $y$  : Total number of species which have bond order less than 1.5.
57. Solution of urea (molar mass = 56 g mol<sup>-1</sup>) boils at 100.18 °C at the atmospheric pressure. If  $K_f$  and  $K_b$  for water are 1.86 and 0.512 K kg mol<sup>-1</sup> respectively, the above solution will freeze at  $-x$  then what is the value of  $x$ ?
58. 3.7 g of a gas at 25 °C occupied the same volume as 0.184 g of hydrogen at 17 °C and at the same pressure. What is the molecular weight of the gas (in g mol<sup>-1</sup>)?
59. The specific conductance of a 0.01 N KCl solution at 23°C is  $0.016 \Omega^{-1} \text{cm}^{-1}$ . The resistance of cell containing the solution at the same temperature was found to be 60  $\Omega$ . The cell constant will be \_\_\_\_\_ cm<sup>-1</sup>.
60. At constant volume, 6 mol of an ideal gas when heated from 300 K to 500 K changes its internal energy by 8000 J. The molar heat capacity at constant volume is \_\_\_\_\_.

### MATHEMATICS

#### Section-A (Multiple Choice Questions)

61. The sum of  $n$  terms of the series

$$\frac{1}{1 \cdot 3 \cdot 5} + \frac{1}{3 \cdot 5 \cdot 7} + \frac{1}{5 \cdot 7 \cdot 9} + \dots \text{ is}$$

(a)  $\frac{n(n+2)}{12(2n-1)(2n+3)}$  (b)  $\frac{n(n+2)}{3(2n+1)(2n+3)}$   
 (c)  $\frac{(n+1)(n+2)}{n(4n^2+1)}$  (d)  $\frac{(n-1)(n+2)}{12(2n+1)(2n+3)}$

62. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is  
 (a)  ${}^8C_3$  (b)  ${}^7C_2$  (c)  ${}^5C_3$  (d)  ${}^8C_2$

63. The constant  $c$  of Rolle's theorem for the function  $f(x) = (x-a)^p(x-b)^q$  in  $[a, b]$  where  $p, q$  are positive integers, is

(a)  $\frac{a+b}{2}$  (b)  $\frac{pa+qb}{p+q}$   
 (c)  $\frac{pb+qa}{p+q}$  (d) None of these

64. If  $z + \frac{1}{z} = \sqrt{3}$ , then  $\sum_{r=1}^4 \left( z^r - \frac{1}{z^r} \right)^2 =$

(a) 8 (b) -8 (c) -12 (d) -13

65. The orthogonal trajectories of the family of semi-cubical parabola  $ay^2 = x^3$  is given by

(a)  $x^2 + 3y^2 = c^2$  (b)  $3x^2 + y^2 = c^2$   
 (c)  $x^2 + 2y^2 = c^2$  (d)  $3y^2 + 2x^2 = c^2$

66. The mean and variance of the data 4, 5, 6, 6, 7, 8,  $x, y$ , where  $x < y$ , are 6 and  $9/4$  respectively. Then  $x^4 + y^2$  is equal to

(a) 162 (b) 320 (c) 674 (d) 420

67. If  $|4 \sin x - 1| < \sqrt{5}$ , then  $x \in$

(a)  $\left[-\pi, \frac{-4\pi}{5}\right) \cup \left(\frac{-\pi}{5}, \frac{\pi}{10}\right] \cup \left(\frac{9\pi}{10}, \pi\right]$   
 (b)  $\left[\frac{-9\pi}{10}, \frac{-\pi}{10}\right) \cup \left(\frac{3\pi}{10}, \frac{7\pi}{10}\right] \cup \left(\pi, \frac{\pi}{10}\right]$   
 (c)  $\left[-\pi, \frac{-9\pi}{10}\right) \cup \left(\frac{-\pi}{10}, \frac{3\pi}{10}\right] \cup \left(\frac{7\pi}{10}, \pi\right]$   
 (d)  $\left(\frac{-7\pi}{10}, \frac{9\pi}{10}\right) \cup \left(\frac{-\pi}{10}, \frac{3\pi}{10}\right)$

68. Let  $I = \int_0^1 |k-x| \cos \pi x dx$ , where  $k$  is any real number.

Then the value of  $I$  for  $k \leq 0$ , is

(a)  $\frac{-2}{\pi^2}$  (b)  $\frac{-3}{\pi^2}$  (c) -2 (d)  $\frac{1}{\pi^2}$

69. If  $\vec{a}, \vec{b}, \vec{c}$  are vectors that do not lie in the same plane and  $\mu$  is a real number then  $[\mu(\vec{a} + \vec{b}) \cdot (\mu^2 \vec{b} \times \mu \vec{c})] = [\vec{a} \cdot ((\vec{b} + \vec{c}) \times \vec{b})]$  for

- (a) no value of  $\mu$   
 (b) exactly one value of  $\mu$   
 (c) exactly two values of  $\mu$   
 (d) exactly three values of  $\mu$

70. A man has a bunch of  $n$  keys, only one of which exactly open the lock. The man tries to open the lock of using the key randomly. The probability that he open the lock at the  $k^{\text{th}}$  attempt by assuming that rejected key has already tried is

(a)  $\frac{k}{n}$  (b)  $\frac{1}{n}$   
 (c)  $\left(1 - \frac{1}{n}\right)^{k-1} \times \frac{1}{n}$  (d) None of these

71. Let  $f(x) = (x+1)^2 - 1, x \geq -1$ . Then the set  $\{x: f(x) = f^{-1}(x)\}$  is

(a)  $\{0, -2\}$  (b)  $\{0, 2, -2\}$   
 (c)  $\{0, -1\}$  (d) None of these

72. Let  $l$  be a positive real number and let

$$A = \begin{bmatrix} 2l-1 & 2\sqrt{l} & 2\sqrt{l} \\ 2\sqrt{l} & 1 & -2l \\ -2\sqrt{l} & 2l & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2l-1 & \sqrt{l} \\ 1-2l & 0 & 2\sqrt{l} \\ -\sqrt{l} & -2\sqrt{l} & 0 \end{bmatrix}$$

If  $\det(\text{adj } A) + \det(\text{adj } B) = 10^6$ , then  $l$  is equal to

(a) 3 (b)  $9/2$  (c) 5 (d) 6

73. For natural numbers  $m, n$  if  $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \dots$ , and  $a_1 = a_2 = 10$ , then  $(m+n)$  is  
 (a) 84 (b) 108 (c) 112 (d) 120

74. The solution of differential equation  $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0 \forall x \in \mathbb{R} - (2n+1)\frac{\pi}{2}, n \in \mathbb{I}$  is

- (a)  $\tan x = \tan y$   
 (b)  $\tan x \tan y = c \forall x \in \mathbb{R}$   
 (c)  $\tan x \tan y = c \forall x \in (\mathbb{R} - \text{odd multiple of } \pi/2)$   
 (d) None of these

75. Let  $f$  be a function from a set  $X$  to a set  $Y$ . Consider the following statements:

P : For each  $x \in X$ , there exists unique  $y \in Y$  such that  $f(x) = y$ .

Q : For each  $y \in Y$ , there exists  $x \in X$  such that  $f(x) = y$ .

R : There exist  $x_1, x_2 \in X$  such that  $x_1 \neq x_2$  and  $f(x_1) = f(x_2)$ .

The negation of the statement " $f$  is one-to-one and onto" is

- (a) P or not R (b) R or not P  
 (c) R or not Q (d) P and not R

76.  $\sum_{r=1}^n \tan^{-1} \left( \frac{2^{r-1}}{1+2^{2r-1}} \right)$  is equal to

- (a)  $\tan^{-1}(2^n)$                       (b)  $\tan^{-1}(2^n) - \frac{\pi}{4}$   
(c)  $\tan^{-1}(2^{n+1})$                     (d)  $\tan^{-1}(2^{n+1}) - \frac{\pi}{4}$
77. The value of  $c$  for which the equation  $2x^2 - 2(2c+1)x + c(c+1) = 0$  may have one root less than  $c$  and other root greater than  $c$  are given by  
(a)  $0 < c < 1$                       (b)  $-1 < c < 0$   
(c)  $c \geq 0$                               (d)  $c > 0$  or  $c < -1$
78. The distance between the line  $\vec{r} = (2\hat{i} + 2\hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j} - 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} + 2\hat{j} + 2\hat{k}) = 10$  is equal to  
(a) 5                      (b) 4                      (c) 3                      (d) 2
79. If chords of contact of tangents from two points  $(x_1, y_1), (x_2, y_2)$  to the ellipse  $\frac{x^2}{52} + \frac{y^2}{13} = 1$  are at right angles, then ratio of the product of abscissas to the ordinates is  
(a)  $-16 : 1$                       (b)  $4 : 1$   
(c)  $16 : 1$                               (d) None of these
80. If  $\int f(x) \cos x \, dx = \frac{1}{2} f^2(x) + c$ , then  $f(x)$  can be  
(a)  $x$                                       (b)  $\sin x$   
(c)  $1$                                       (d)  $\cos x$

### Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

81. The inclination of the straight line passing through the point  $A(3, 6)$  and the mid-point of the line joining the points,  $B(4, 4)$  and  $C(-2, 4)$ , with  $x$ -axis (in degrees) is \_\_\_\_\_.

82. If the angle between the line  $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$  and the plane  $x + 2y + 3z = 4$  is  $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$ , then  $\lambda$  equals \_\_\_\_\_.
83. Let  $f(x) = x^2 + bx + 7$ . If  $f'(5) = 2f'\left(\frac{7}{2}\right)$ , then the value of  $|b|$  is \_\_\_\_\_.
84. If  $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$ , then the value of  $\cos 4x$  is  $\frac{-42}{9\lambda}$ . The value of  $\lambda$  is \_\_\_\_\_.
85. The difference between degree and order of a differential equation that represents the family of curves given by  $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$ ,  $a > 0$  is \_\_\_\_\_.
86. Consider the A.P.  $a_1, a_2, \dots, a_n, \dots$  and the G.P.  $b_1, b_2, \dots, b_n, \dots$  such that  $a_1 = b_1 = 1$ ;  $a_9 = b_9$  and  $\sum_{r=1}^9 a_r = 369$ , then  $b_7 =$  \_\_\_\_\_.
87. The number of integer values of  $m$ , for which the  $x$ -coordinate of the point of intersection of the lines  $3x + 4y = 11$  and  $y = mx + 1$  is also an integer, is \_\_\_\_\_.
88. The value of  $\lim_{x \rightarrow 0} \frac{ex}{e - (1+x)^x}$  is \_\_\_\_\_.
89. If the system of linear equations  $x + ky + 3z = 0$ ;  $3x + ky - 2z = 0$ ;  $2x + 4y - 3z = 0$  has a non-zero solutions  $(x, y, z)$ , then  $\frac{6xz}{3y^2}$  is equal to \_\_\_\_\_.
90. If  $\int \frac{\cos^4 x \, dx}{\sin^3 x (\sin^5 x + \cos^5 x)^{3/5}} = -\frac{1}{A} \left( \frac{1 + \tan^5 x}{\tan^5 x} \right)^B + K$ , then the value of  $A + 5B$  is \_\_\_\_\_.

SPACE FOR ROUGH WORK