JEE Main

CLASS XII

Physics : Electric Charges and Fields; Electrostatic; Potential and Capacitance; Current Electricity

Chemistry : The Solid State; Solutions; Electrochemistry; Chemical Kinetics; Surface Chemistry

Mathematics : Relations and Functions; Inverse Trigonometric Functions; Matrices; Determinants

PART TEST - 1

Instructions:

I. The JEE Main Chapterwise Test Paper consists of one paper containing 60 objective questions (four options with single correct answer) and 30 numerical value type questions from the above mentioned chapters of Physics, Chemistry and Mathematics.
II. The duration of paper would be 3 hours (180 minutes).
III. There will be total 90 questions : Physics - 30, Chemistry - 30, Mathematics - 30.
IV. Each question will carry 4 marks. For each correct response the applicant will be awarded four marks. For each incorrect answer there will be deduction of one mark.
V. There will be no negative marking for unattended questions. More than one answer of single question will also be considered as incorrect response and will be negatively marked.

Max. Marks : 300 **Time :** 180 minutes

PHYSICS

Section-A (Multiple Choice Questions)

1. Two conducting spheres of radii 3 cm and 1 cm are separated by a distance of 10 cm in free space. If the spheres are charged to same potential of 10 V each, the force of repulsion between them is

(a) $\left(\frac{1}{3}\right)\times10^{-9}$ N (b) $\left(\frac{2}{9}\right)\times10^{-9}$ N

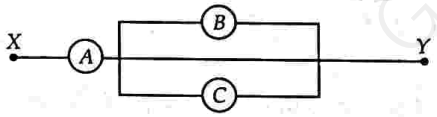
(c) $\left(\frac{1}{9}\right)\times10^{-9}$ N (d) $\left(\frac{4}{3}\right)\times10^{-9}$ N

2. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d ($d << l$) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them

(a) $v \propto x^{-1/2}$ (b) $v \propto x^{-1}$

(c) $v \propto x^{1/2}$ (d) $v \propto x$

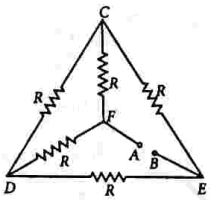
3. A, B and C are voltmeters of resistance R, $1.5R$ and $3R$ respectively. When some potential difference is applied between X and Y, the voltmeter readings are V_A, V_B and V_C respectively. Then



(a) $V_A = V_B = V_C$ (b) $V_A \neq V_B = V_C$

(c) $V_A = V_B \neq V_C$ (d) $V_A \neq V_B \neq V_C$

4. Five equal resistances, each of resistance R, are connected as shown in figure below. A battery of V volt is connected between A and B. The current flowing in FC will be



(a) $\frac{3V}{R}$ (b) $\frac{V}{R}$ (c) $\frac{V}{2R}$ (d) $\frac{2V}{R}$

5. The net flux of the uniform electric field $E = 3\times10^3\,\hat{i}$ N C^{-1} through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes is

(a) 60 N C^{-1} m^2 (b) 120 N C^{-1} m^2

(c) 240 N C^{-1} m^2 (d) 0 N C^{-1} m^2

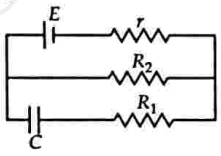
6. In a quark model of elementary particles, a neutron is made of one up quark of charge $\frac{2}{3}e$ and two down quark of charges $\left(-\frac{1}{3}e\right)$. If they have a triangle configuration with side length of the order of 10^{-15} m. The electrostatic potential energy of neutron in MeV is

(a) 7.68 (b) −5.21 (c) −0.48 (d) 9.34

7. Electric charge is uniformly distributed along a long straight wire of radius 1 mm. The charge per cm length of the wire is Q coulomb. Another cylindrical surface of radius 50 cm and length 1 m symmetrically encloses the wire. The total electric flux passing through the cylindrical surface is

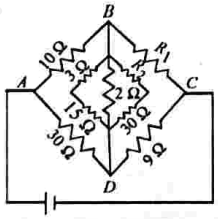
(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{100Q}{\varepsilon_0}$ (c) $\frac{10Q}{\pi\varepsilon_0}$ (d) $\frac{100Q}{\pi\varepsilon_0}$

8. The charge on the capacitor of capacitance C shown in the figure will be



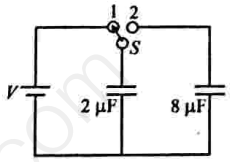
(a) CE (b) $\frac{CER_1}{R_1+r}$ (c) $\frac{CER_2}{R_2+r}$ (d) $\frac{CER_1}{R_2+r}$

9. In the Wheatstone's bridge shown below, in order to balance the bridge we must have



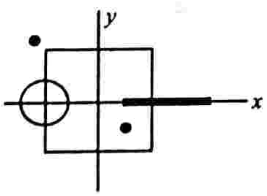
(a) $R_1 = 3\ \Omega, R_2 = 3\ \Omega$
(b) $R_1 = 6\ \Omega, R_2 = 1.5\ \Omega$
(c) $R_1 = 1.5\ \Omega$, R_2 = any finite value
(d) $R_1 = 3\ \Omega$, R_2 = any finite value

10. A 2 μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(a) 0%
(b) 20%
(c) 75%
(d) 80%

11. A disk of radius $a/4$ having a uniformly distributed charge 6 C is placed in the x-y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8 C is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges −7 C and 3 C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2, y = \pm a/2, z = \pm a/2$. The electric flux through this cubical surface is



(a) $\frac{-2\,C}{\varepsilon_0}$ (b) $\frac{2\,C}{\varepsilon_0}$ (c) $\frac{10\,C}{\varepsilon_0}$ (d) $\frac{12\,C}{\varepsilon_0}$

12. Given below are two statements :

Statement I : When a charge q is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement II : The electric field at a distance $r(r < R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

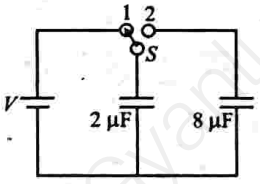
Choose the correct answer from the options given below.

(a) Both statement-I and statement-II are true.
(b) Both statement-I and statement-II are false.
(c) Statement-I is true but statement-II is false.
(d) Statement-I is false but statement-II is true.

13. In a potentiometer experiment, when three cells A, B and C are connected in series the balancing length is found to be 740 cm. If A and B are connected in series balancing length is 440 cm and for B and C connected in series that is 540 cm. Then the emf of E_A, E_B and E_C are respectively (in volts)

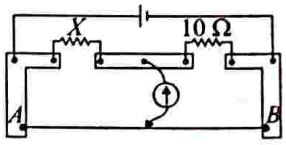
(a) 1, 1.2 and 1.5 (b) 1, 2 and 3
(c) 1.5, 2 and 3 (d) 1.5, 2.5 and 3.5

14. A 2 μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(a) 0% (b) 20% (c) 75% (d) 80%

15. A meter bridge is set-up as shown, to determine an unknown resistance X using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is

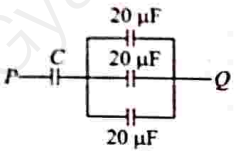


(a) 10.2 ohm (b) 10.6 ohm
(c) 10.8 ohm (d) 11.1 ohm

16. Two cells with the same emf E and different internal resistances r_1 and r_2 are connected in series to an external resistance R. The value of R so that the potential difference across the first cell be zero is

(a) $\sqrt{r_1 r_2}$ (b) $r_1 + r_2$ (c) $r_1 - r_2$ (d) $\frac{r_1 + r_2}{2}$

17. If the equivalent capacitance between P and Q of the combination of the capacitors shown in figure is 30 μF, the capacitor C is

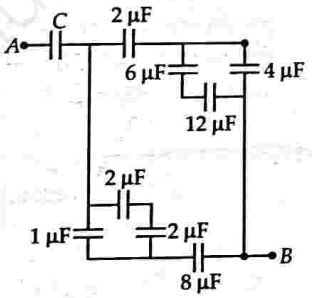


(a) 60 μF (b) 30 μF
(c) 10 μF (d) 5 μF

18. When two resistances R_1 and R_2 are connected in series, they consume 12 W power. When they are connected in parallel, they consume 50 W power. What is the ratio of the powers of R_1 and R_2?
(a) 1/4 (b) 4 (c) 3/2 (d) 3

19. Capacitor C_1 of capacitance 1 mF and capacitor C_2 of capacitance 2 mF are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time $t = 0$. Then
(a) the current in each of the two discharging circuit is zero at $t = 0$
(b) the current in the two discharging circuits at $t = 0$ are equal but not zero
(c) the current in the two discharging circuits at $t = 0$ are unequal
(d) C_1 loses 50% of its initial charge sooner than C_2 loses 50% its initial charge.

20. In the given network, the value of C, so that an equivalent capacitance between A and B is 3 μF, is

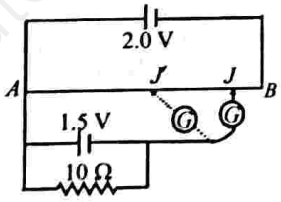


(a) $\frac{1}{5}$ μF
(b) $\frac{31}{5}$ μF
(c) 48 μF
(d) 36 μF

Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

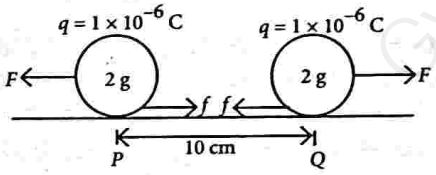
21. The figure below shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in the open circuit is 75 cm. When a resistor of 10 Ω is used in the external circuit of the cell, the balance point shifts to 65 cm length of potentiometer wire. The internal resistance of the cell (in ohm) is _____.



22. A photographic flash unit consists of a xenon filled tube. It gives a flash of average power 2000 W for 0.04 s. The flash is due to discharge of a fully charged capacitor of 40 μF. The voltage (in kV) to which it is charged before a flash is _____.

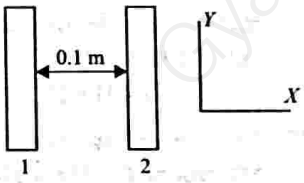
23. Figure shows the portions of two infinite parallel non-conducting sheets having the magnitude of the surface charge densities $\sigma_{(+)} = 6.8\ \mu C/m^2$ and $\sigma_{(-)} = 4.3\ \mu C/m^2$ for the positively and negatively charged sheets, respectively. The electric field E between the sheets is $x \times 10^5\ N\ C^{-1}$. The value of x. is _____.

24. Two particles each having a mass of 2 g and charge 1×10^{-6} C are in limiting equilibrium on a horizontal surface with a separation of 10 cm between them. The coefficient of friction between each particle and the surface is same. The value of this coefficient of friction is _____.



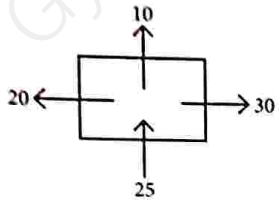
25. An electric toaster uses nichrome for its heating element. When a negligibly small current passes through it, its resistance at room temperature (27.0°C) is found to be 75.3 Ω. When the toaster is connected to a 230 V supply, the current settles, after a few seconds to a steady value of 2.68 A. The temperature coefficient of resistance is $1.7 \times 10^{-4}\ °C^{-1}$. The steady temperature (in degree celsius) of the nichrome element is _____.

26. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (*i.e.* plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. Its speed is $x \times 10^6$ m/s when it hits plate 2. The value of x is _____.
($e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg)



27. An electric field $\vec{E} = 200\hat{i}$ N/C for $x > 0$ and $\vec{E} = -200\hat{i}$ N C^{-1} for $x < 0$ exists. A right circular cylinder of length 20 cm and radius 5 cm has its center at the origin and its axis along the x-axis, so that one face is at $x = +10$ cm and the other is at $x = -10$ cm. The net charge inside the cylinder is 2.78×10^{-n} C. Then the value of n is _____.
(Take $\varepsilon_0 = 8.854 \times 10^{-12}$ SI units).

28. Flux (in SI units coming out and entering a closed surface is shown in the figure. The charge (in coulomb) enclosed by the closed surface is $x\varepsilon_0$ where the value of x is _____.



29. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $K_2 = 6$ and thickness $2d/3$. Now, the new capacitance (in pF) of the capacitor is _____.

30. Three resistances P, Q, R each of 2 Ω and an unknown resistance S form the four arms of a Wheatstone's bridge circuit. When a resistance of 6 Ω is connected in parallel to S, the bridge gets balanced. Then, the value of S (in Ω) is _____.

CHEMISTRY

Section-A (Multiple Choice Questions)

31. Which one of the following has a different crystal lattice from those of the rest?
(a) Ag (b) V (c) Cu (d) Pt

32. The K_H value (kbar) of argon(I), carbon dioxide(II) formaldehyde(III) and methane(IV) are respectively 40.3, 1.67, 1.83×10^{-5} and 0.413 at 298 K. The increasing order of solubility of gas in liquid is
(a) I < II < IV < III (b) III < IV < II < I
(c) I < III < II < IV (d) I < IV < II < III

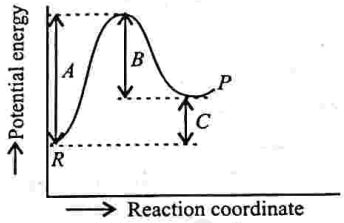
33. Given below are two statements:
Statement-I : The limiting molar conductivity of KCl (strong electrolyte) is higher as compared to that of CH_3COOH (weak electrolyte).
Statement-II : Molar conductivity decreases with decrease in concentration of electrolyte.
In the light of the above statements, choose the most appropriate answer from the options given below:
(a) Statement-I is false but statement-II is true.
(b) Both statement-I and statement-II are true.
(c) Statement-I is true but statement-II is false.
(d) Both statement-I and statement-II are false.

34. In a catalytic conversion of N_2 to NH_3 by Haber's process, the rate of reaction expressed as change in the concentration of ammonia per unit time is 40×10^{-3} mol L^{-1} s^{-1}. If there are no side reactions, the rate of the reaction expressed in terms of hydrogen is
(a) 60×10^{-3} mol L^{-1} s^{-1} (b) 20×10^{-3} mol L^{-1} s^{-1}
(c) 1200 mol L^{-1} s^{-1} (d) 10.3×10^{-3} mol L^{-1} s^{-1}

35. The limiting molar conductivities Λ° for NaCl, KBr and KCl are 126, 152 and 150 S cm^2 mol^{-1} respectively. The Λ° for NaBr is
(a) 128 S cm^2 mol^{-1} (b) 176 S cm^2 mol^{-1}
(c) 278 S cm^2 mol^{-1} (d) 302 S cm^2 mol^{-1}

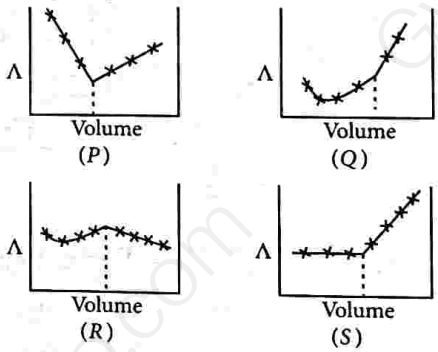
36. The rate expression for the reaction, $A_g + B_g \longrightarrow C_g$ is rate = $kC_A^2 C_B^{1/2}$. What changes in the initial concentrations of A and B will cause the rate of reaction to increase by a factor of eight?
(a) $C_A \times 2$; $C_B \times 2$ (b) $C_A \times 2$; $C_B \times 4$
(c) $C_A \times 1$; $C_B \times 4$ (d) $C_A \times 4$; $C_B \times 1$

37. The potential energy diagram for a reaction $X \rightarrow Y$ is given. A and C in the graph correspond to



(a) $A \rightarrow$ activation energy, $C \rightarrow \Delta H^\circ$
(b) $A \rightarrow$ energy of reactants, $C \rightarrow$ energy of products
(c) $A \rightarrow \Delta H^\circ$, $C \rightarrow$ activation energy
(d) $A \rightarrow$ activation energy, $C \rightarrow$ threshold energy

38. $AgNO_{3(aq)}$ was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance (Λ) versus the volume of $AgNO_3$ is



(a) P (b) Q
(c) R (d) S

39. B^- ions form a close packed structure. If the radius of B^- ion is 200 pm then the cation (A^+) having radii 88 pm can fit into
(a) tetrahedral hole (b) octahedral hole
(c) both of these (d) none of these.

40. For a binary solution of acetone and aniline which of the following set of conditions is true?

(a) $\Delta_{mix}H < \Delta_{mix}H_{(ideal)}$; $\Delta_{mix}S < \Delta_{mix}S_{(ideal)}$
$\Delta_{mix}G < 0$; $p_A < p_{A(ideal)}$

(b) $\Delta_{mix}H < \Delta_{mix}H_{(ideal)}$; $\Delta_{mix}S < \Delta_{mix}S_{(ideal)}$
$\Delta_{mix}G > 0$; $p_A > p_{A(ideal)}$

(c) $\Delta_{mix}H > \Delta_{mix}H_{(ideal)}$; $\Delta_{mix}S < \Delta_{mix}S_{(ideal)}$
$\Delta_{mix}G < 0$; $p_A < p_{A(ideal)}$

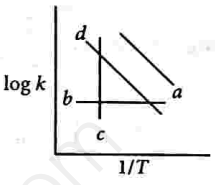
(d) $\Delta_{mix}H < \Delta_{mix}H_{(ideal)}$; $\Delta_{mix}S > \Delta_{mix}S_{(ideal)}$
$\Delta_{mix}G < 0$; $p_A > p_{A(ideal)}$

41. An aqueous solution of X is added slowly to an aqueous solution of Y as shown in List I. The variation in conductivity of these reactions is given in List II. Match List I with List II and select the correct answer using the code given below the lists :

	List I		List II
P.	$(C_2H_5)_3N$ + CH_3COOH (X) (Y)	1.	Conductivity decreases and then increases
Q.	KI (0.1M) + $AgNO_3$ (0.01 M) (X) (Y)	2.	Conductivity decreases and then does not change much
R.	CH_3COOH + KOH (X) (Y)	3.	Conductivity increases and then does not change much
S.	NaOH + HI (X) (Y)	4.	Conductivity does not change much and then increases

	P	Q	R	S
(a)	3	4	2	1
(b)	4	3	2	1
(c)	2	3	4	1
(d)	1	4	3	2

42. Consider the following plots of rate constant (k) versus $\frac{1}{T}$ for four different reactions. Which of the following orders is correct for the activation energies of these reactions?



(a) $E_b > E_a > E_d > E_c$ (b) $E_a > E_c > E_d > E_b$
(c) $E_b > E_d > E_c > E_a$ (d) $E_c > E_a > E_d > E_b$

43. **Statement 1:** According to Freundlich, $\frac{x}{m} = k.P^{1/n}$.

Statement 2 : The isotherm shows variation of the amount of gas adsorbed by the adsorbent with temperature.

(a) Both statement 1 and statement 2 are true.
(b) Both statement 1 and statement 2 are false.
(c) Statement 1 is true but statement 2 is false.
(d) Statement 1 is false but statement 2 is true.

44. The volume of gases H_2, HCl, CO_2 and NH_3 adsorbed by 1 g of activated charcoal at 298 K are in the order

(a) $H_2 > CO_2 > HCl > NH_3$
(b) $H_2 > HCl > CO_2 > NH_3$
(c) $NH_3 > CO_2 > HCl > H_2$
(d) $NH_3 > HCl > CO_2 > H_2$

45. Given below are two statements :

Statement-I : Frenkel defects are vacancy as well as interstitial defects.

Statement-II : Frenkel defect leads to colour in ionic solids due to presence of F-centres.

Choose the most appropriate answer for the statements from the options given below:

(a) Statement-I is false but statement-II is true.
(b) Both statement-I and statement-II are false.
(c) Statement-I is true but statement-II is false.
(d) Both statement-I and statement-II are true.

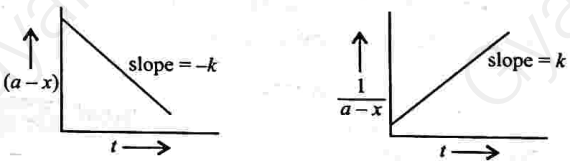
46. The elevation in boiling point of a solution of 13.44 g of $CuCl_2$ in 1 kg of water using the following information will be (Molecular weight of $CuCl_2$ = 134.4 and $K_b = 0.52$ K molal^{-1})

(a) 0.16 (b) 0.05 (c) 0.1 (d) 0.02

47. The correct order of equivalent conductance at infinite dilution of LiCl, NaCl and KCl is

(a) LiCl > NaCl > KCl (b) KCl > NaCl > LiCl
(c) NaCl > KCl > LiCl (d) LiCl > KCl > NaCl

48. Two plots are shown below between concentration and time (t). Which of the given orders are shown by the graphs respectively?



(a) Zero order and first order
(b) First order and second order
(c) Zero order and second order
(d) First order and first order

49. Suppose the mass of a single Ag atom is 'm'. Ag metal crystallizes in *fcc* lattice with unit cell of length 'a'. The density of Ag metal in terms of 'a' and 'm' is

(a) $\frac{4m}{a^3}$ (b) $\frac{2m}{a^3}$ (c) $\frac{m}{a^3}$ (d) $\frac{m}{4a^3}$

50. When the same amount of solute 'P' and 'Q' are separately dissolved in 500 g water, the ΔT_f values are 0.15 K and 0.30 K respectively. If the molecular weight of 'P' is 80 g mol^{-1}, then the molecular weight of 'Q' is

(a) 30 g mol^{-1} (b) 60 g mol^{-1}
(c) 40 g mol^{-1} (d) 45 g mol^{-1}

Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

51. If density of solid NaCl is 43.1 g cm^{-3}, then the distance between Na^+ and Cl^- ions is _____ $\times 10^{-10}$ m. (Nearest integer)
(Given : $N_A = 6.02 \times 10^{23}$ mol^{-1})

52. How many faradays are required to reduce 1 mol of $Cr_2O_7^{2-}$ to Cr^{3+} in acid medium?

53. The vapour pressure of a liquid solution containing *A* and *B* is 99 torr. Calculate mole % of *B* in vapour phase. (Nearest integer) (Given : $p_A^\circ = 100$ torr, $p_B^\circ = 80$ torr)

54. If 75% of a first order reaction was completed in 90 minutes, 40% of the same reaction would be completed in approximately (in minutes) _____.
(Take : log 2 = 0.30; log 2.5 = 0.40)

55. The gold number of gelatin is 0.01. Calculate the amount of gelatin (in mg) to be added to 1000 mL of a colloidal sol of gold to prevent its coagulation, before adding 1 mL of 10% NaCl solution.

56. The molal freezing point constant of water is 1.86 K m^{-1}. If 342 g of cane sugar ($C_{12}H_{22}O_{11}$) is dissolved in 1000 g of water, the solution will freeze at _____ °C.

57. The equivalent conductivity of a solution containing 2.54 grams of $CuSO_4$ per litre is 91.0 Ω^{-1} cm^2 eq^{-1}. Its conductivity would be _____ $\times 10^{-3}$ Ω^{-1} cm^{-1}.

58. An organic compound undergoes first-order decomposition. The time taken for decomposition to 1/8 and 1/10 of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively. What is the value of $\frac{[t_{1/8}]}{[t_{1/10}]} \times 10$?
(Take $\log_{10}2 = 0.3$)

59. For the coagulation of 500 mL of arsenious sulphide sol, 2 mL of 1M NaCl is required. What is the flocculation value of NaCl?

60. Metal *M* crystallises into a *fcc* lattice with the edge length of 3.0×10^{-8} cm. The atomic mass of the metal is _____ g/mol. (Nearest integer)
(Use : $N_A = 6.02 \times 10^{23}$ mol^{-1},
density of metal, $M = 9.03$ g cm^{-3})

MATHEMATICS

Section-A (Multiple Choice Questions)

61. Let $f: R \to R$ defined by
$f(x) = 2x^3 + 2x^2 + 300x + 5 \sin x$, then *f* is
(a) one-one onto (b) one-one into
(c) many-one onto (d) many-one into

62. If $S_k = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$, $k \in N$, where *N* is the set of all natural numbers, then $(S_2)^n (S_k)^{-1}$, for $n \in N$, equals to
(a) S_{2n-k} (b) S_{2n+k-1}
(c) S_{2n+k-2} (d) S_{2n+k+1}

63. $\theta = \tan^{-1}(2\tan^2\theta) - \tan^{-1}\left\{\left(\frac{1}{3}\right)\tan\theta\right\}$, if
(a) $\tan\theta = -2$ (b) $\tan\theta = 3$
(c) $\tan\theta = 1$ (d) Both (a) and (c)

64. The value of '*a*' for which $f: R \to R$, $f(x) = \frac{ax+7}{x^2+4}$ is invertible, is
(a) 0 (b) −1
(c) 1 (d) any value of *a*

65. If $[x]$ denotes the greatest integer $\le x$, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$, $[\cot\theta]x + y = 0$
(a) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.
(b) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.
(c) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.
(d) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

66. If $\tan^{-1}\left(\frac{1}{1+1\cdot2}\right) + \tan^{-1}\left(\frac{1}{1+2\cdot3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \tan^{-1} x$, then the value of *x* is
(a) *n* (b) $\frac{n+1}{2}$ (c) $\frac{n}{n+2}$ (d) $\frac{n}{n+1}$

67. If $A(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{pmatrix}$, then $A(\theta)^3$ will be a null matrix if and only if
(a) $\theta = (2k+1)\pi/3, k \in I$ (b) $\theta = (4k-1)\pi/3$ $k \in I$
(c) $\theta = (3k-1)\pi/4, k \in I$ (d) none of these

68. Let $f: R - \left\{\frac{4}{3}\right\} \to R$, $f(x) = \frac{4x+7}{3x-4}$.
Let $f_2(x) = f(f(x)), f_3(x) = f(f_2(x)), \ldots, f_n(x) = f(f_{n-1}(x))$.

Then $f_{2008}(x) + f_{2009}(x) =$

(a) $\frac{3x^2+7}{3x-4}$ (b) $\frac{x^2+7}{3x-4}$

(c) $\frac{2x^2-7}{3x-4}$ (d) $\frac{x^2-7}{3x-4}$

69. Let t be a positive integer and

$\Delta_t = \begin{vmatrix} 2t-1 & m^2-1 & \cos^2(m^2) \\ {}^mC_t & 2^m & \cos^2(m) \\ 1 & m+1 & \cos(m^2) \end{vmatrix}$, then the value of

$\sum_{t=0}^{m} \Delta_t$ is equal to

(a) 2^m (b) 0

(c) $2^m \cos^2(2^m)$ (d) m^2

70. The value of $\lim_{n\to\infty} \sum_{k=2}^{n} \cos^{-1}\left(\frac{1+\sqrt{(k-1)k(k+1)(k+2)}}{k(k+1)}\right)$ is

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

71. Let Z be the set of integers. If

$A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$ is

(a) 2^{18} (b) 2^{12} (c) 2^{15} (d) 2^{10}

72. Let $a = \lim_{x\to 1}\left(\frac{x}{\ln x} - \frac{1}{x \ln x}\right)$; $b = \lim_{x\to 0} \frac{x^3-16x}{4x+x^2}$;

$c = \lim_{x\to 0} \frac{\ln(1+\tan x)}{x}$ and

$d = \lim_{x\to -1} \frac{(x+1)^3}{3(\sin(x+1)-(x+1))}$, then the determinant of matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

(a) -6 (b) -2 (c) 0 (d) -4

73. If $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right) = \alpha$, then x^2 is equal to

(a) $\sin\alpha$ (b) $\cos 2\alpha$ (c) $\cos\alpha$ (d) $\sin 2\alpha$

74. If $f(\alpha) = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix}$, then $f(\sqrt[3]{3})$ is equal to

(a) 1 (b) -4 (c) 4 (d) 2

75. If $g(x) = x^2 + x - 1$ and $(gof)(x) = x^2 + 3x + 1$, then $f(2)$ is equal to

(a) 5 (b) 3 (c) 1 (d) 6

76. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 3 & -1 & 9 \end{pmatrix}$, then the value of det (adj (adj A)) equals

(a) 11 (b) 121

(c) 1331 (d) 14641

77. The value of $\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

(a) $\frac{26}{25}$ (b) $\frac{25}{26}$

(c) $\frac{50}{51}$ (d) $\frac{52}{51}$

78. For a positive integer n, if

$\Delta = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$

then $\frac{\Delta}{(n!)^3} - 4$ is divisible by

(a) n (b) $n+1$

(c) $n+2$ (d) none of these

79. If the trace of the matrix

$A = \begin{pmatrix} x-5 & 0 & 2 & 4 \\ 3 & x^2-10 & 6 & 1 \\ -2 & 3 & x-7 & 1 \\ 1 & 2 & 0 & -2 \end{pmatrix}$ assumes the value zero, then the value of x equals to

(a) $-6, -4$ (b) $-6, 4$

(c) $6, 4$ (d) $6, -4$

80. The value of the determinant $\begin{vmatrix} {}^xC_1 & {}^xC_2 & {}^xC_3 \\ {}^yC_1 & {}^yC_2 & {}^yC_3 \\ {}^zC_1 & {}^zC_2 & {}^zC_3 \end{vmatrix}$ equals to

(a) $\frac{(x-y)(y-z)(z-x)}{6}$

(b) $\frac{(xyz)(x-y)(y-z)(z-x)}{12}$

(c) $\frac{(x-y)(y-z)(z-x)}{12}$

(d) $(xyz)(x-y)(y-z)(z-x)$

Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

81. If $\cos^{-1}(4x^3 - 3x) = a + b\cos^{-1} x$ for $-1 < x < -\frac{1}{2}$. Then $|[a + b + 2]|$ = ___ where $[\cdot]$ is G.I.F.

82. Let $f: R \to R$ be a function defined by

$$f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)(2+x^{25})\right)^{1/50}.$$

If the function $g(x) = f(f(f(x))) + f(f(x))$, then the greatest integer less than or equal to $g(1)$ is _____.

83. If $A=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$, for $0 < x < \frac{\pi}{2}$ and $A + A^T = I$, then the value of $\sec x$ is _____.

84. Let $A=\begin{bmatrix}l & m & n\\ p & q & r\\ 1 & 1 & 1\end{bmatrix}$ and $B = A^2$.

If $(l-m)^2 + (p-q)^2 = 9$, $(m-n)^2 + (q-r)^2 = 16$, $(n-l)^2 + (r-p)^2 = 25$, then the value of det (B) equals _____.

85. If $f(x)=\left(100^5 - x^{10}\right)^{\frac{1}{10}}$, then the value of $\frac{1}{2^{10}} f(f(1024))$ is _____.

86. If $\sin^{-1} x + \sin^{-1} y = \pi$ and, if $x = \lambda y$, then the value of λ must be _____.

87. Given $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ if $f(\theta)\, f(\phi) = kf(\theta + \phi)$. Then the value of k is _____.

88. If $A=\begin{bmatrix}1 & -1 & 1\\ 0 & 2 & -3\\ 2 & 1 & 0\end{bmatrix}$, $B = (\text{adj } A)$ and $C = 5A$, then the value of $\frac{|\text{adj } B|}{|C|}$ is _____.

89. Let $f: R \to [1, \infty)$ be defined as $f(x)=\log_{10}(\sqrt{3x^2 - 4x + k + 1} + 10)$.
If $f(x)$ is surjective, then $3k =$_____.

90. For $k \in R$, let the solutions of the equation $\cos(\sin^{-1}(x\cot(\tan^{-1}(\cos(\sin^{-1}x))))) = k$, $0 < |x| < \frac{1}{\sqrt{2}}$ be α and β, where the inverse trigonometric functions take only principal values. If the solutions of the equation $x^2 - bx - 5 = 0$ are $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ and $\frac{\alpha}{\beta}$, then $\frac{b}{k^2}$ is equal to _____.

SPACE FOR ROUGH WORK