JEE Main

CLASS XII

Physics : Moving Charges and Magnetism; Magnetism and Matter; Electromagnetic Induction; Alternating Current

Chemistry : General Principles and Processes of Isolation of Elements; The *p*-Block Elements (Group 15 to 18); The *d*- & *f*-Block Elements; Coordination Compounds

Mathematics : Continuity and Differentiability; Application of Determinants; Integrals

PART TEST - 2

Instructions:

I. The JEE Main Chapterwise Test Paper consists of one paper containing 60 objective questions (four options with single correct answer) and 30 numerical value type questions from the above mentioned chapters of Physics, Chemistry and Mathematics.
II. The duration of paper would be 3 hours (180 minutes).
III. There will be total 90 questions : Physics - 30, Chemistry - 30, Mathematics - 30.
IV. Each question will carry 4 marks. For each correct response the applicant will be awarded four marks. For each incorrect answer there will be deduction of one mark.
V. There will be no negative marking for unattended questions. More than one answer of single question will also be considered as incorrect response and will be negatively marked.

Max. Marks : 300 **Time :** 180 minutes

PHYSICS

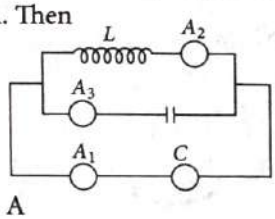
Section-A (Multiple Choice Questions)

1. Two identical magnetic dipoles of magnetic moment 2 A m^2 are placed at a separation of 2 m with their axis perpendicular to each other in air. The resultant magnetic field at a midpoint between the dipoles is
   (a) $4\sqrt{5}\times10^{-5}$ T (b) $2\sqrt{5}\times10^{-5}$ T
   (c) $4\sqrt{5}\times10^{-7}$ T (d) $2\sqrt{5}\times10^{-7}$ T

2. An electron and an alpha particle having equal velocities enter at right angles to the magnetic field. In this field
   (a) both the particles move on the circular path of the same radius
   (b) both the particles move in straight path
   (c) the radius of the path of alpha particle is greater than that of the electron
   (d) the radius of the path of alpha particle is less than that of the electron

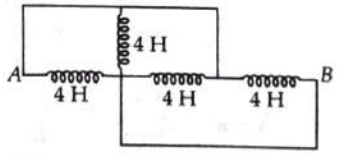
3. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is
   (a) $\pi\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$ (c) $2\pi\sqrt{LC}$ (d) $\sqrt{LC}$

4. For the circuit shown in fig. the ammeter A_2 reads 1.6 A and ammeter A_3 reads 0.4 A. Then
   (a) $\omega_0 = \frac{4}{\sqrt{LC}}$
   (b) $f_2 = \frac{2\pi}{\sqrt{LC}}$
   (c) the ammeter A_1 reads 1.2 A
   (d) the ammeter A_1 reads 2 A



5. A cyclotron's oscillator frequency is 10 MHz. If the radius of the dees is 60 cm, the kinetic energy (in MeV) of the proton beam produced by the accelerator is ($e = 1.6 \times 10^{-19}$ C, $m_P = 1.6 \times 10^{-27}$ kg)
   (a) 5 MeV (b) 6 MeV (c) 7 MeV (d) 8 MeV

6. The equivalent inductance between A and B is



   (a) 1 H (b) 4 H (c) 0.8 H (d) 16 H

7. The magnetic field at the point of intersection of diagonals of a square wire loop of side L carrying a current I is

(a) $\frac{\mu_0 I}{\pi L}$ (b) $\frac{2\mu_0 I}{\pi L}$

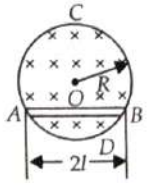
(c) $\frac{\sqrt{2}\mu_0 I}{\pi L}$ (d) $\frac{2\sqrt{2}\mu_0 I}{\pi L}$

8. A copper wire of length 50.0 cm and total resistance of $1.1 \times 10^{-2}\ \Omega$ is formed into a circular loop and placed perpendicular to a uniform magnetic field that is increasing at the constant rate of 10.0 mT/s. At what rate is thermal energy generated in the loop?

(a) 1.32×10^{-8} W (b) 2.36×10^{-4} W

(c) 3.68×10^{-6} W (d) 4.23×10^{-5} W

9. A uniform magnetic field of field induction B fills a cylindrical volume of radius R. A rod AB of length $2l$ is placed as shown in figure. If B is changed at the rate dB/dt, the emf that is produced by the changing magnetic field and that acts between the ends of rod is



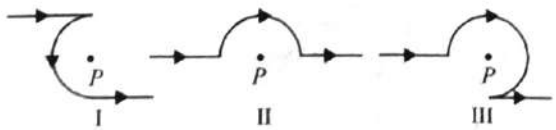
(a) $\frac{dB}{dt} l\sqrt{R^2 - l^2}$ (b) $\frac{dB}{dt} l\sqrt{R^2 + l^2}$

(c) $\frac{1}{2}\frac{dB}{dt} l\sqrt{R^2 - l^2}$ (d) $\frac{1}{2}\frac{dB}{dt} l\sqrt{R^2 + l^2}$

10. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio $\frac{T'}{T}$ is

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$.

11. The magnetic field B at the centre of a circular coil of radius r is π times that due to a long straight wire at a distance r from it, for equal currents. Figure here shows three cases: in all cases the circular part has radius r and straight ones are infinitely long. For same current the magnetic field B at the centre P in cases I, II, III have the ratio



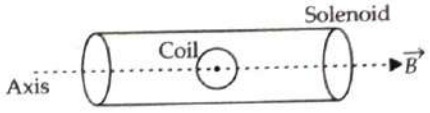
(a) $\left(\frac{-\pi}{2}\right) : \frac{\pi}{2} : \left(\frac{3\pi}{4} - \frac{1}{2}\right)$

(b) $\left(\frac{-\pi}{2} + 1\right) : \left(\frac{\pi}{2} + 1\right) : \left(\frac{3\pi}{4} + \frac{1}{2}\right)$

(c) $\frac{-\pi}{2} : \frac{\pi}{2} : \frac{3\pi}{4}$

(d) $\left(\frac{-\pi}{2} - 1\right) : \left(\frac{\pi}{2} - \frac{1}{4}\right) : \left(\frac{3\pi}{4} + \frac{1}{2}\right)$

12. The torque required to hold a small circular coil of 10 turns, area 1 mm^2 and carrying a current of $\left(\frac{21}{44}\right)$A in the middle of a long solenoid of 10^3 turns/m carrying a current of 2.5 A, with its axis perpendicular to the axis of the solenoid is



(a) 1.5×10^{-6} N m (b) 1.5×10^{-8} N m

(c) $1.5 \times 10^{+6}$ N m (d) $1.5 \times 10^{+8}$ N m

13. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating, it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to

(a) induction of electrical charge on the plate.

(b) shielding of magnetic lines of force as aluminium is a paramagnetic material.

(c) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

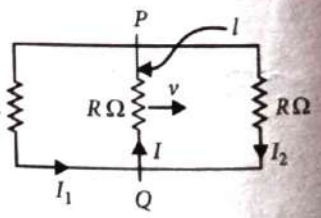
(d) development of air current when the plate is placed.

14. In an AC circuit, V and I are given by $V = 150\sin(150t)$ volt and $I = 150\sin\left(150t + \frac{\pi}{3}\right)$ ampere. The power dissipated in the circuit is

(a) 106 W (b) 150 W

(c) 5625 W (d) zero

15. A rectangular loop has a sliding connector PQ of length l and resistance $R\ \Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(a) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

(b) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(c) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(d) $I_1 = I_2 = I = \frac{Blv}{R}$

16. A solenoid of inductance 50 mH and resistance 10 Ω is connected to a battery of 6 V. The time elapsed before the current acquires half of its steady state value is

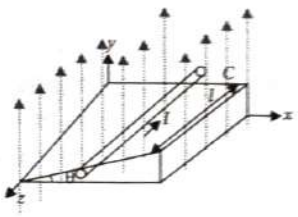
(a) 2 ms (b) 3.5 ms (c) 5 ms (d) 5.5 ms

17. An AC source is of $\frac{200}{\sqrt{2}}$ volt, 50 hertz. The value of voltage after $\frac{1}{600}$ second from the start is

(a) 200 volt (b) $\frac{200}{\sqrt{2}}$ volt

(c) 100 volt (d) 50 volt

18. A conducting wire of length l and mass m is placed on two inclined rails as shown in the figure. A current I is flowing in the wire in the direction shown. When no magnetic field is present in the region, the wire is just on the verge of sliding. When a vertically upward magnetic field is switched on, the wire starts moving up the incline. The distance travelled by the wire as a function of time t will be



(a) $\frac{1}{2}\left[\frac{IBl}{m}-2g\right]t^2$

(b) $\frac{1}{2}\left[\frac{IBl}{m}\times\frac{1}{\cos\theta}-2g\sin\theta\right]t^2$

(c) $\frac{1}{2}\left[\frac{IBl}{m}-2g\sin\theta\right]t^2$

(d) $\frac{1}{2}\left[\frac{IBl}{m}\frac{\cos2\theta}{\cos\theta}-2g\sin\theta\right]t^2$

19. An ideally efficient transformer has a primary power input of 10 kW. The secondary current when the transformer is on load is 25 ampere. If the ratio of primary to secondary turns is 8 : 1, then the potential difference applied to the primary coil is

(a) $\frac{10^4\times8^2}{25}$ V (b) $\frac{10^4\times8}{25}$ V

(c) $\frac{10^4}{25\times8^2}$ V (d) $\frac{10^4}{25\times8}$ V

20. A small square loop of wire of side l is placed inside a large square loop of wire of side $L(L >> l)$. The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to

(a) l/L (b) l^2/L (c) L/l (d) L^2/l

Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

21. A series combination of resistor (R), capacitor (C) is connected to an A.C. source of angular frequency 'ω'. Keeping the voltage same, if the frequency is changed to $\frac{\omega}{3}$, the current becomes half of the original current. Then the capacitive reactance is _____ times of the resistance at the former frequency.

22. A horizontal telegraph wire 2.5 km long running east and west is a part of a circuit whose resistance is 35 Ω. The wire falls to the ground from a height of 10 m. If $g = 9.8$ m s^{-2} and $B_h = 2 \times 10^{-5}$ T, then the current induced (in A) in the circuit is_____.

23. A short magnet of moment 6.75 Am^2 produces a neutral point on its axis. If horizontal component of Earth's magnetic field is 5×10^{-5} Wb/m^2, then the distance of the neutral point (in cm) from it is_____.

24. A particle of charge -16×10^{-18} coulomb moving with velocity 10 m s^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electric field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is 1×10^x Wb/m^2. The value of x is _____.

25. A thin insulated wire forms a plane spiral of $N = 100$ tight turns carrying a current $i = 8$ mA. The radii of inside and outside turns are equal to $a = 50$ mm and $b = 100$ mm. The magnetic moment (in A m^2) of the spiral with the given current is _____.

26. The coefficient of mutual-inductance of two circuits A and B is 3 mH and their respective resistances are 10 ohm and 4 ohm. Current (in ampere) should change in 0.02 second in the circuit A, so that the induced current in B should be 0.006 ampere, is_____.

27. A bar magnet has a pole strength of 3.6 A m, magnetic length of 12 cm and cross-sectional area of 0.90 cm^3. The nearest integral value of magnetic intensity (in A m^{-1}) at the centre of the magnet is_____.

28. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. The resistance in ohms needed to be connected in series with the coil in order that each division reads 1 volt is _____.

29. A wire of length 10 cm translates in a direction making an angle of 60° with its length. The plane of motion is perpendicular to a uniform magnetic field of 1.0 T that exists in the space. The emf induced (in V) between the ends of the rod if the speed of translation is 20 cm/s is _____.

30. A 50 W, 100 V lamp is to be connected to an AC mains of 200 V, 50 Hz. The capacitance (in μF) essential to be put in series with the lamp is _____.

CHEMISTRY

Section-A (Multiple Choice Questions)

31. Function of potassium ethyl xanthate in froth floatation process is to make the ore
(a) hydrophobic (b) hydrophilic
(c) heavier (d) lighter.

32. The number of P-O-P bridges in the structure of phosphorus pentoxide and phosphorus trioxide are respectively
(a) 6, 6 (b) 5, 5 (c) 5, 6 (d) 6, 5

33. The atomic numbers of Vanadium (V), Chromium (Cr), Manganese (Mn) and Iron (Fe) are respectively 23, 24, 25 and 26. Which one of these may be expected to have the highest second ionisation enthalpy?
(a) V (b) Cr (c) Mn (d) Fe

34. Match List-I (Complex ions) with List-II (Number of unpaired electrons) and select the correct answer using the codes given ahead the lists:

List-I (Complex ions)		List-II (Number of unpaired electrons)	
(A)	$[CrF_6]^{4-}$	(P)	One
(B)	$[MnF_6]^{4-}$	(Q)	Two
(C)	$[Cr(CN)_6]^{4-}$	(R)	Three
(D)	$[Mn(CN)_6]^{4-}$	(S)	Four
		(T)	Five

(a) A → S, B → P, C → Q, D → T
(b) A → Q, B → T, C → R, D → P
(c) A → S, B → T, C → Q, D → P
(d) A → Q, B → P, C → R, D → T

35. Which series of reactions correctly represents chemical relations related to iron and its compound?
(a) $Fe \xrightarrow{O_2,\ heat} Fe_3O_4 \xrightarrow{CO,\ 600\ ^\circ C} FeO \xrightarrow{CO,700\ ^\circ C} Fe$
(b) $Fe \xrightarrow{dil.\ H_2SO_4} FeSO_4 \xrightarrow{H_2SO_4,\ O_2} Fe_2(SO_4)_3 \xrightarrow{heat} Fe$
(c) $Fe \xrightarrow{O_2,\ heat} FeO \xrightarrow{dil.\ H_2SO_4} FeSO_4 \xrightarrow{heat} Fe$
(d) $Fe \xrightarrow{Cl_2,\ heat} FeCl_3 \xrightarrow{heat,\ air} FeCl_2 \xrightarrow{Zn} Fe$

36. Match the column I with column II and mark the appropriate choice.

Column I		Column II	
(A)	Thiosulphuric acid	(i)	H_2SO_5
(B)	Caro's acid	(ii)	$H_2S_2O_6$
(C)	Marshall's acid	(iii)	$H_2S_2O_3$
(D)	Dithionic acid	(iv)	$H_2S_2O_8$

(a) (A) → (i); (B) → (ii); (C) → (iii); (D) → (iv)
(b) (A) → (iv); (B) → (iii); (C) → (ii); (D) → (i)
(c) (A) → (iii); (B) → (i); (C) → (iv); (D) → (ii)
(d) (A) → (ii); (B) → (iii); (C) → (i); (D) → (iv)

37. Among the following ion which one has the highest paramagnetism?
(a) $[Cr(H_2O)_6]^{3+}$ (b) $[Fe(H_2O)_6]^{2+}$
(c) $[Cu(H_2O)_6]^{2+}$ (d) $[Zn(H_2O)_6]^{2+}$

38. If $\Delta_o < P$, the correct electronic configuration for d^4 system will be
(a) $t_{2g}^4\ e_g^0$ (b) $t_{2g}^3\ e_g^1$
(c) $t_{2g}^0\ e_g^4$ (d) $t_{2g}^2\ e_g^2$

39. The shape of XeF_5^- will be
(a) square pyramidal
(b) trigonal bipyramidal
(c) pentagonal planar
(d) pentagonal bipyramidal.

40. The "spin-only" magnetic moment [in units of Bohr magneton, (μ_B)] of Ni^{2+} in aqueous solution would be (atomic number of Ni = 28)
(a) 2.84 (b) 4.90 (c) 0 (d) 1.73

41. The correct order of energies of *d*-orbitals of metal ion in a square planar complex is
(a) $d_{xy} = d_{yz} = d_{zx} > d_{x^2-y^2} = d_{z^2}$
(b) $d_{x^2-y^2} = d_{z^2} > d_{xy} = d_{yz} = d_{zx}$
(c) $d_{x^2-y^2} > d_{z^2} > d_{xy} > d_{yz} = d_{zx}$
(d) $d_{x^2-y^2} > d_{xy} > d_{z^2} > d_{zx} = d_{yz}$

42. Sodium cyanide is added as a depressant in the froth floatation process when the ore contains a mixture of ZnS and PbS. This is because
(a) $Pb(CN)_2$ gets precipitated without any effect on ZnS
(b) ZnS forms soluble complex while PbS forms froth
(c) PbS forms soluble complex while ZnS forms froth
(d) $Zn(CN)_2$ gets precipitated without any effect on PbS.

43. Which of the following increasing order is not correct as mentioned in the property with it?
(a) $HClO < HClO_2 < HClO_3 < HClO_4$ (thermal stability)
(b) $HClO_4 < HClO_3 < HClO_2 < HClO$ (oxidising power)
(c) $F^- < Cl^- < Br^- < I^-$ (reducing nature)
(d) $HIO_4 < ICl < I_2 < HI$ (oxidation number of iodine)

44. Which of the following arrangements does not represent the correct order of the property stated against it?
(a) Sc < Ti < Cr < Mn : number of oxidation states

(b) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour

(c) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: ionic size

(d) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: stability in aqueous solution.

45. Given below are two statements :

Statement I : Zincite is a sulphide ore of zinc and copper glance is a sulphide ore of copper.

Statement II : It is possible to separate two sulphide ores by adjusting proportion of oil to water or by using 'depressants' in a froth floatation method.

Choose the most appropriate answer from the options given below.

(a) Statement I is true but statement II is false.

(b) Both statement I and statement II are true.

(c) Both statement I and statement II are false.

(d) Statement I is false but statement II is true.

46. Match List-I with List-II.

List - I (Species)		List - II (No. of lone pair of electrons on the central atom)	
(A)	XeF_2	(i)	0
(B)	XeO_2F_2	(ii)	1
(C)	XeO_3F_2	(iii)	2
(D)	XeF_4	(iv)	3

Choose the most appropriate answer from the options given below.

(a) (A)-(iii), (B)-(iv), (C)-(ii), (D)-(i)

(b) (A)-(iv), (B)-(ii), (C)-(i), (D)-(iii)

(c) (A)-(iii), (B)-(ii), (C)-(iv), (D)-(i)

(d) (A)-(iv), (B)-(i), (C)-(ii), (D)-(iii)

47. **Statement-1 :** $[Fe(H_2O)_5NO]SO_4$ is paramagnetic.

Statement-2 : The Fe in $[Fe(H_2O)_5NO]SO_4$ has three unpaired electrons.

(a) Statements-1 and 2 are true and statement-2 is a correct explanation for statement-1.

(b) Statements-1 and 2 are true and statement-2 is not a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false

(d) Statement-1 is false, statement-2 is true.

48. The colour of $KMnO_4$ is due to

(a) $L \rightarrow M$ charge transfer transition

(b) $\sigma \rightarrow \sigma^*$ transition

(c) $M \rightarrow L$ charge transfer transition

(d) $d - d$ transition.

49. Identify the correct trend given below :
(Atomic no. : Ti = 22, Cr = 24 and Mo = 42)

(a) Δ_o of $[Cr(H_2O)_6]^{2+} > [Mo(H_2O)_6]^{2+}$ and Δ_o of $[Ti(H_2O)_6]^{3+} > [Ti(H_2O)_6]^{2+}$

(b) Δ_o of $[Cr(H_2O)_6]^{2+} > [Mo(H_2O)_6]^{2+}$ and Δ_o of $[Ti(H_2O)_6]^{3+} < [Ti(H_2O)_6]^{2+}$

(c) Δ_o of $[Cr(H_2O)_6]^{2+} < [Mo(H_2O)_6]^{2+}$ and Δ_o of $[Ti(H_2O)_6]^{3+} > [Ti(H_2O)_6]^{2+}$

(d) Δ_o of $[Cr(H_2O)_6]^{2+} < [Mo(H_2O)_6]^{2+}$ and Δ_o of $[Ti(H_2O)_6]^{3+} < [Ti(H_2O)_6]^{2+}$

50. From the Ellingham graphs on carbon, which of the following statements is false?

(a) CO_2 is more stable than CO at less than 983 K.

(b) CO reduces Fe_2O_3 to Fe at less than 983 K.

(c) CO is less stable than CO_2 at more than 983 K.

(d) CO reduces Fe_2O_3 to Fe in the reduction zone of Blast furnace.

Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

51. In the complex acetylbromidodicarbonyl*bis* (triethylphosphine)iron(II), the number of Fe—C bond(s) is _____.

52. Find the number of metals from the given metals which can be commercially purified by zone refining methods. Si, Ge, Ga, Al, Ti, Zr

53. How many orbitals are involved in the hybridisation of iodine in IF_7?

54. How many of the following metals can be refined by vapour phase refining ?
Zn, Zr, Hg, Cd, Ni, Ti, Co, Pt, Fe

55. Find the number of reaction from the given reactions which can show calcination process :

(i) $CaCO_3.MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$

(ii) $CuCO_3.Cu(OH)_2 \xrightarrow{\Delta} 2CuO + H_2O + CO_2$

(iii) $2Cu_2S + 3O_2 \xrightarrow{\Delta} 2Cu_2O + 2SO_2$

56. $EDTA^{4-}$ is ethylenediaminetetraacetate ion. The total number of N—Co—O bonds in $[Co(EDTA)]^{1-}$ complex ion is _____.

57. Calculate the spin only magnetic moment (in B.M.) of Ce^{3+} ion.

58. Silver ore is oxidised by using oxygen from air as follows :

$4Ag + xNaCN + 2H_2O + O_{2(air)} \rightarrow yNa[Ag(CN)_2] + 4NaOH$

What is the value of $(x - y)$?

59. How many moles of $K_2Cr_2O_7$ are reduced by one mole of Sn^{2+} ion ?

60. The number of water molecule(s) directly bonded to the metal centre in $CuSO_4 \cdot 5H_2O$ is _____.

MATHEMATICS

Section-A (Multiple Choice Questions)

61. The point of extremum of the function

$g(x) = \int_{\pi/2}^{x} e^{-t^2}(4-t^2)\,dt$ is/are

(a) $x = 0$ (b) $x = 2, -2$

(c) $x = 1$ (d) None of these

62. $\int \left(x^{3m} + x^{2m} + x^m\right)\left(2x^{2m} + 3x^m + 6\right)^{\frac{1}{m}} dx =$

(a) $\frac{1}{6m}(2x^{3m} + 3x^{2m} + 6x^m)^{1/m} + c$

(b) $\frac{1}{6(m+1)}(2x^{3m} + 3x^{2m} + 6x^m)^{(m+1)/m} + c$

(c) $\frac{1}{6m}(2x^{3m} + 3x^{2m} + 6x^m)^{(m+1)/m} + c$

(d) $\frac{1}{6(m+1)}(2x^{3m} + 3x^{2m} + 6x^m)^{1/m} + c$

63. If $x^{mx^{mx^{mx^{\cdots^{\infty}}}}} = y^{ny^{ny^{ny^{\cdots^{\infty}}}}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{dy}{dx} = \frac{m-n}{m+n}\left(\frac{x}{y}\right)$ (b) $\frac{m+n}{m-n}\left(\frac{y}{x}\right)$

(c) $\frac{mx}{ny}$ (d) $\frac{my}{nx}$

64. The value of $\frac{(5050)\int_0^1 (1-x^{50})^{100}\,dx}{\int_0^1 (1-x^{50})^{101}\,dx}$ is

(a) 5049 (b) 5051

(c) 5050 (d) none of these

65. If $f(x) = \int_x^{x^2} \frac{dt}{(\log t)^2}$, $x \neq 0, x \neq 1$, then $f(x)$ is

(a) strictly increasing in (1, 2)

(b) strictly decreases in (2, ∞)

(c) strictly increases in (2, ∞)

(d) strictly decreases in (0, 3)

66. If $f_n(x) = e^{f_{n-1}(x)}$, $\forall\ n \in N$ and $f_0(x) = x$, then $f_n'(x)$ equals

(a) $\prod_{m=1}^{m=n} f_m(x)$ (b) $\prod_{m=1}^{m=n} f_m'(x)$

(c) $\sum_{m=1}^{m=n} f_m(x^2)$ (d) None of these

67. If a be the digit at unit's place in $11^{2012} + 23^{2012} - 3^{2012}$, then $\int_{a-1}^{a} \frac{dx}{\sqrt{1-x^2} - x + \frac{1}{x}} =$

(a) $\frac{\pi}{9}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

68. If the function f given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0$ $(x \neq 1)$ is

(a) 7 (b) −7 (c) 5 (d) 6

69. Let $f(x) = |[x]\,x|$ for $-1 \le x \le 2$, then

(a) $f(x)$ is discontinuous at $x = 0$

(b) $f(x)$ is differentiable at $x = 1$

(c) $f(x)$ is not differentiable at $x = 2$

(d) $f(x)$ is differentiable at $x = 2$

70. The minimum value of $\left(1 + \frac{1}{\sin^n x}\right)\left(1 + \frac{1}{\cos^n x}\right)$ is

(a) $(1 + 2^n)^2$ (b) $(1 + 2^{n/2})^2$

(c) 2 (d) 1

71. If $f(x) = \frac{\sin 4\pi[\pi^2 x]}{7 + [x]^2}$, $[\cdot]$ denotes the greatest integer function, then $f(x)$ is

(a) continuous $\forall\ x$, but $f'(x)$ does not exist.

(b) discontinuous at some x.

(c) $f''(x)$ exist $\forall\ x$

(d) $f'(x)$ exist but $f''(x)$ does not exist for some values of x.

72. $\int \sin 2x\, d(\tan x)$ is equal to

(a) $2\log|\cos x| + C$ (b) $\log|\cos x| + C$

(c) $2\log|\sec x| + C$ (d) $\log|\sec x| + C$

73. If $f(x) = \left(\frac{A^2-1}{A^2+1}\right)x^3 - 3x + e^{\pi/2}$ is a non-increasing function of x in R then the set of possible values of A[which does not depend on x] is

(a) $[1, \infty)$ (b) $[-1, 1]$

(c) $(-\infty, \infty)$ (d) $(-\infty, -1]$

74. Let $f(x) = [\tan x] + \sqrt{\tan x - [\tan x]}$ $\forall\ 0 \le x < \frac{\pi}{2}$, where $[\cdot]$ denotes the greatest integer function, then

(a) $f(x)$ is discontinuous at $x = \frac{\pi}{4}$

(b) $f(x)$ is continuous at $x = 0$ & $\frac{\pi}{4}$

(c) $f(x)$ is discontinuous at $x = 0$

(d) $f(x)$ has infinite points of discontinuity.

75. $\int_0^{\sqrt{3}} \frac{dx}{[x+\sqrt{x^2+1}]^3} =$

(a) $\frac{27}{2}$ (b) $\frac{8\sqrt{3}}{27}$

(c) $8\sqrt{3}$ (d) $8\sqrt{3}-\frac{27}{2}$

76. If OT and ON are perpendiculars drawn from the origin to the tangent and normal to the curve $x = a\sin^3 t$, $y = a\cos^3 t$ an arbitrary point, then $\frac{4OT^2+ON^2}{a} =$

(a) a^2 (b) a

(c) a^3 (d) None of these

77. Let $J_{n,m} = \int_0^{1/2} \frac{x^n}{x^m-1}dx, \forall\, n > m$ and $n, m \in N$.

Consider a matrix $A = [a_{ij}]_{3\times 3}$, where

$a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}$. Then $|A^{-1}|$ is

(a) $\frac{1}{210\times 2^{20}}$ (b) 210×2^{20}

(c) 210×2^{16} (d) 210×2^{18}

78. If $I_1 = \int_x^1 \frac{1}{1+z^2}dz$ and $I_2 = \int_1^{1/x} \frac{1}{1+z^2}dz$, $\forall\, x \to 0$ and $g(x) = \frac{I_1}{I_2}$, then $g'(x)$ equals, $\forall\, x \in R^+$

(a) 1 (b) -1

(c) 0 (d) None of these

79. If $\int \frac{dx}{x(x^{11}+1)} = \frac{1}{n}\log\left(\frac{x^{11}}{x^{11}+1}\right)+c$, then value of n is

(a) 11 (b) 8 (c) 4 (d) 3

80. If $y = \sqrt{x+\sqrt{y+\sqrt{x+\sqrt{y+.....\infty}}}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{y^2+x}{2y^3-2xy-1}$ (b) $\frac{y^2}{2y^3-2xy-1}$

(c) $\frac{y^2-x}{2y^3-2xy-1}$ (d) $\frac{2y^3-2xy-1}{y^2-x}$

Section-B (Numerical Value Type)

Attempt any 5 questions out of 10

81. Let $f: R \to R$ be a function defined as

$f(x) = \begin{cases} 3\left(1-\frac{|x|}{2}\right), & \text{if } |x| \le 2 \\ 0, & \text{if } |x| > 2 \end{cases}$.

Let $g: R \to R$ be given by $g(x) = f(x+2) - f(x-2)$. If n and m denote the number of points in R, where g is not continuous and not differentiable, respectively, then $n + m$ is equal to _____.

82. If $\int \frac{\cos\left(\theta+\frac{\pi}{4}\right)d\theta}{\sqrt{\sin 2\theta+3\sin^2 2\theta+\sin^3 2\theta}}$

$= k\sec^{-1}\left\{2\cos^2\left(\frac{\pi}{4}-\theta\right)\right\}+c$, then $\frac{1}{k^2} =$_____.

83. If $lx + my = 1$ touches the curve $(ax)^n + (by)^n = 1$, then $\left(\frac{l}{a}\right)^{\frac{n}{n-k}} + \left(\frac{m}{b}\right)^{\frac{n}{n-k}} = 1$, then the value of k is _____.

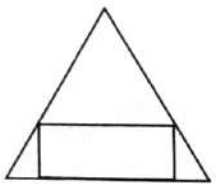
84. If $\int \frac{(1+x^n)^{\frac{1}{n}}}{x^{n+2}}dx = a\left(1+\frac{1}{x^4}\right)^b + c$, then $a + b =$ _____.

85. If the function $f(x) = x^3 - 6x^2 + ax + b$ satisfies Rolle's theorem in the interval $[0, 3]$ and $f'(3) = 0$, then $a =$ _____.

86. If the function $f(x) = \ln\left(\frac{1-x+x^2}{1+x+x^2}\right)$ is decreasing in $\left(-\frac{\alpha}{3}, \frac{\alpha}{3}\right)$, then greatest integral value of α is _____.

87. If $\int \cos^4 x\,dx = Ax + B\sin 2x + C\sin 4x + D$, then the value of $8A + 4B + 32C$ is _____.

88. In an equilateral triangle of side length $\sqrt{2}$, a rectangle is inscribed as shown in the figure, then the largest area of such a rectangle is _____ sq. units.



89. If $\int_0^{\pi/2} \sqrt{\sin\phi}\cos^5\phi\,d\phi = \frac{m}{n}$, then $\frac{m+n}{n-m}$ is equal to _____.

90. Let $f(x) = |x - 2|$ and $g(x) = f(f(x))$. Then derivative of g at the point $x = 5$ is _____.